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Original scientific paper

DYNAMIC MODELLING AND ASYMPTOTIC POINT STABILIZATION CONTROL OF TWO DIFFERENTIAL WHEELED MOBILE ROBOT

Drilon Bunjaku[1], Jovan Stefanovski[2], Mile Stankovski[3]

[1]Informatics Engineering, FMCE at University of Mitrovica, Republic of Kosovo
[2]Control & Informatics Div., JP "Streževo", Bitola, Republic of Macedonia
[3]Faculty of Electrical Engineering and Information Technologies,
"Ss. Cyril and Methodius" University in Skopje,
Rugjer Bošković bb, P.O. box 574, 1001 Skopje, Republic of Macedonia
drilon.bunjaku@uni-pr.edu

A b s t r a c t: In this paper, we present the dynamic modelling of two differential wheeled mobile robot, and also propose an easily implementable control strategy, for stabilizing the nonlinear and nonholonomic WMR system around the desired final posture. The asymptotic stability is approached by using two PI controllers. The dynamic model of WMR is used in the simulation environment of Matlab/Simulink, for testing the proposed stabilizing control strategy. The validity of control strategy is verified by the simulation results.

Key words: wheeled mobile robot; dynamic modelling; nonholonomic robots; stabilization control; linear controller

ДИНАМИЧКО МОДЕЛИРАЊЕ И КОНТРОЛА НА ПРИБЛИЖУВАЊЕТО КОН АСИМПТОТСКАТА СТАБИЛНОСТ НА МОБИЛЕН РОБОТ СО ДВЕ ДИФЕРЕНЦИЈАЛНИ ТРКАЛА

А п с т р а к т: Во овој труд е претставено динамичко моделирање на мобилен робот со две диференцијални тркала и предложена е стратегија за контрола која може лесно да се имплементира за стабилизирање на нелинеарниот и нехолономичен систем на мобилен робот со тркала околу посакуваната крајна положба. Приближувањето кон асимптотската стабилност е изведено со два ПИ контролера. Динамичниот модел на мобилниот робот со тркала е искористен во симулациската околина на Matlab/Simulink за тестирање на предложената стратегија за контрола на стабилизацијата. Валидноста на стратегијата е верификувана со резултатите од симулацијата.

Клучни зборови: мобилен робот со тркала; динамичко моделирање; нехолономични роботи; контрола на стабилизација; линеарен контролер

1. INTRODUCTION

Nowadays, the complexity and hazardous of working process and working environment in many fields have been increased. Consequently, as a result, the interest of manipulation and mobile robots application have been exponentially increased last few years as well (for further details consult [1], [2]). We may find an application of mobile robot in fields like mining, military, medicine, aerospace, industry, under water, etc., which require high accuracy, responsibility, and reliability. Therefore, wheeled mobile robots as a particular category of mobile robot are widely studied during the last decade, because of their simplicity and applicability. Wheeled mobile robots (WMR) are wheeled vehicles or platforms which are supposed to navigate from the initial point toward the desired or final point in an autonomous manner.

The mobile robot has visual, proximity, positioning and object detection capabilities [3].

The performance of the WMR depends on many factors, like types of sensors and actuators, their sensitivities and limitations, but mainly it depends on the robustness of the designed controller. Controllers should be fast responsive and immune to the disturbances and parameter variations.

There are many research papers focused on designing the control of a wheeled mobile robot. Depending on the configuration of the robot, many of them have proposed a controller which track the desired trajectory in the most effective way in 2D space [4]–[6]. Mostly include the kinematic model of WMR, while very few include dynamic model due to the complexity of the model and high nonlinearity degree.

However, because of the center-of-gravity (COG) shifts and load changes caused by large loads and the serious nonlinear friction at the high speed, the accuracy of the path-tracking decreases and the robots stray from the predefined path, which clearly increases the danger of hitting obstacles. Therefore, motion control is one of the most fundamental topics for mobile robots [7].

The navigation problem of mobile robots could be separated into four basic problems:

1. Obstacle avoidance,
2. Autonomous trajectory generation (path planning),
3. Trajectory tracking,
4. Point stabilization.

All the afore mentioned navigation problems ought to use localization sensors system. In [8] authors presented the impact of using the dead reckoning sensors on the improvement of positioning accuracy of GPS and DGPS in application of land vehicles. The road construction vehicles, farm vehicles and mining vehicles require accuracy of the order of a few centimetres. Hence, carrier phase differential GPS (CP-DGPS) technology provides such requirement. In [9] a nonlinear velocity independent control law has been designed for the farm tractor (relies upon the kinematic model) to perform both curved paths and straight lines following by using a CP-DGPS sensor. The GPS is limited for the indoor mobile robots application with high accuracy requirements. Therefore, the indoor GPS system with fix beacons could be used.

In the following text we group the cited papers by the separation, thus for the first and second navigation problem: in [10], authors elaborated a technique of constructing (generating) a feasible trajectory for WMR by assembling arcs of a simple curves, and extended the research by adding fuzzy logic control for obstacle avoidance.

In [11] authors analyzed the controllability of the nonholonomic multibody robots with inequality constrained, and proposed an algorithm for generating path planning based on a bitmap discretisation.

In [12] authors presented an algorithm for generating a trajectory by using simple arcs and straight lines. Furthermore, achieving obstacle avoidance through the composition of trajectories based on the set of configuration sub-goals that lead to collision-free motion.

This paper is confined to the trajectory tracking and the point-stabilization for WMRs moving/operating in the 2D real-world space, within the respective separation of navigation problem.

– Regarding the third navigation problem: In [13] a new kinematical control method, named Lyapunov-based Guidance Control (LGC), has been proposed for the trajectory tracking of nonholonomic WMRs. Through the application of back stepping methodology, in [14] is proposed a control scheme for trajectory tracking for the considered augmented model including kinematics and dynamics of the mobile robot.

In [6] authors proposed higher order sliding motion control based on the kinematic model for tracking the trajectory, the outcome results were satisfactory but it requires highly processing power compared to existing control methods. In [7] a digital acceleration control method is proposed for the path-tracking of a wheeled mobile robot to deal with COG shifts and load changes.

In [15] authors presented dynamic modelling of the WMR by using Lagrange formalism, and proposed two motion control laws for dynamic object tracking by using Lyapunov direct method and computed torque method.

– Regarding the fourth navigation problem: In [16] the Point Stabilization of Mobile Robots is achieved by using Nonlinear Model Predictive Control.

In [17] authors elaborated a method for posture stabilization of the wheeled mobile robot by using a hybrid automata-based controller.

In [18] authors extended the nonholonomic integrated model by double integrating it, because it fails to capture the cases where both kinematic and dynamic of WMR are taken into account. Then, logic-based hybrid controller was proposed that yields global stability and convergence of the closed-loop system to an arbitrarily small neighborhood of the origin.

Motivated by the scientific approaches which are used in aforementioned works, the problem of interest in this paper is to design a stabilizing control about a desired posture. In such a way, that it will bring WMR to navigate from initial posture to the predefined desired posture, and solve the problem of asymptomatic stability. Besides the existing methods, the novelty of this paper is the simplicity of understanding, and easily implementable in the practical real slow-speed operating WMR.

The organization of the paper is as follows. In Section 2 it is presented the kinematic modelling of the robot. Continuously, the elaboration of dynamic modelling of two differential wheeled mobile robot is given in Section 3. In Section 4 is presented the proposed control strategy for solving the problem of point stabilization, and it is followed by subsections of robot position control and robot orientation control. The simulation results for the proposed control system design are given in Section 5. The conclusion remarks are given is section 6.

Remarks on the notation. Matrices are denoted by upper-case letters, and vectors and scalars are denoted by lower-case letters.

2. KINEMATIC MODELLING

The number of possible wheeled mobile robots realizations is almost infinite, depending on the number, type, implementation, geometric characteristics, and motorization of the wheels [19].

The mobile robot in this paper is driven by two independent differential wheels, and one free-wheel or caster wheel for balancing the platform.

The robot posture in Cartesian space x, y, θ will be described by the global reference coordinate frame $\{0\}$.

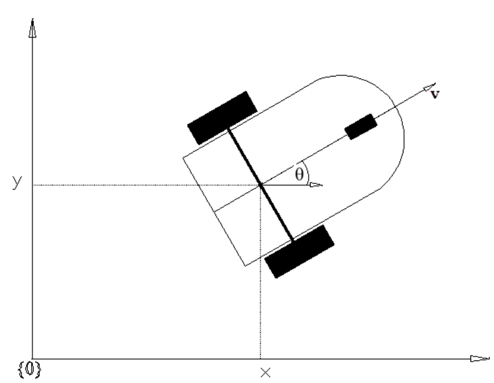

Fig. 1. WMR in 2D Cartesian space

Before proceeding with kinematic model some assumptions will be defined:

1) Both motors produce the same torque;

2) There is no friction on wheels or pure rolling without slipping;

3) The distribution of mass is uniform;

4) The robot will run on a flat surface, meaning the potential energy is zero;

5) There is no deformation on wheels or terrain.

The robot posture in Cartesian coordinate frame is specified by the generalized coordinate vectors $\mathbf{q}_B = [x_B, y_B, \theta]^T$ or $\mathbf{q}_G = [x_G, y_G, \theta]^T$. The point B is represented by x_B and y_B, which is the center of the wheel axis, while by x_G and y_G is represented the center of gravity of the platform. The distance between the center of the wheel axis B and the center of gravity G of the platform is denoted by l, while heading of point G with respect to point B defined the orientation angle θ of the platform.

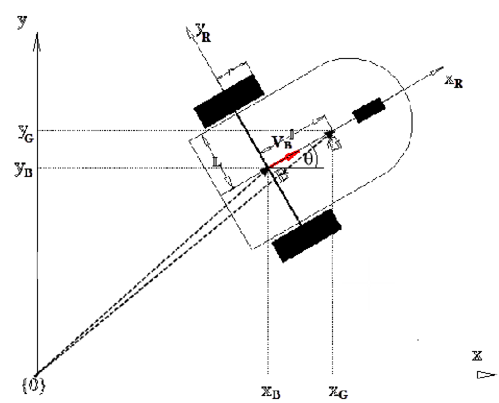

Fig. 2. Generalized coordinate vectors q_B , q_G in R^3

Linear velocity of the right respectively left wheel could be expressed as a function of it is angular velocity $v_r = r\dot{\theta}_r$ and $v_l = r\dot{\theta}_l$, where r is the radius of the driving wheel.

Based on the condition of assumption 2, the linear and angular velocity at point B could be expressed by the following

$$v = \frac{(v_r + v_l)}{2} = \frac{r\dot{\theta}_r + r\dot{\theta}_l}{2}, \quad (1)$$

$$\omega = \frac{(v_r - v_l)}{2L} = \frac{r\dot{\theta}_r - r\dot{\theta}_l}{2L}. \quad (2)$$

In matrix form:

$$\begin{bmatrix} \upsilon \\ \omega \end{bmatrix} = \begin{bmatrix} \frac{r}{2} & \frac{r}{2} \\ \frac{r}{2L} & -\frac{r}{2L} \end{bmatrix} \begin{bmatrix} \dot{\theta}_r \\ \dot{\theta}_l \end{bmatrix} = D \begin{bmatrix} \dot{\theta}_r \\ \dot{\theta}_l \end{bmatrix} \tag{3}$$

The position of center of gravity G could be described by the global reference frame {0} in a vectorial form, in a complex plane:

$$x_q + \vec{j}\ y_q = (x_b + \vec{j}\ y_b) + l(\cos\theta + \vec{j}\sin\theta) = = (x_b + \vec{j}\ y_b) + le^{j\theta}$$

$$\overrightarrow{OG} = \overrightarrow{OB} + \overrightarrow{BG} = \overrightarrow{OB} + le^{j\theta}. \tag{4}$$

If we differentiate the equation (4), the velocity relations could be found:

$$\overrightarrow{\upsilon_G} = \overrightarrow{\upsilon_B} + \vec{j} l \dot{\theta} e^{j\theta} \tag{5}$$

$$\overrightarrow{\upsilon_G} = \dot{x}_B + \dot{y}_G \tag{6}$$

$$\overrightarrow{\upsilon_B} = \dot{x}_B + \dot{y}_B = \upsilon e^{j^\theta} \tag{7}$$

Through the substitution of (7) in (5), it is possible to express the velocity of point G in terms of the general linear υ and angular $\dot{\theta}$ velocities:

$$\overrightarrow{\upsilon_G} = \upsilon e^{j\theta} + \vec{j} l \dot{\theta} e^{j\theta} = (\upsilon + \vec{j} l \theta) e^{j\theta} \tag{8}$$

The velocity of point G could be expressed in term of the real and imaginary part, by approaching the substitution of (6) in (8):

$$\begin{aligned} \dot{x}_G &= \upsilon\cos\theta - l\omega\sin\theta \\ \dot{y}_G &= \upsilon\sin\theta + l\omega\cos\theta \end{aligned}. \tag{9}$$

Wheeled mobile platforms are subject to nonintegrable kinematic constraints, known as nonholonomic constraints (17). The nonholonomic constraint could be defined by eliminating the parameter υ from equation (9):

$$\dot{y}_G\cos\theta - \dot{x}_G\sin\theta - l\omega = 0 \tag{10}$$

From equation (10) for $\theta = 0$ the velocity in y direction is zero, $\dot{y}_G = 0$, while for $\theta = \frac{\pi}{2}$ the velocity in x direction is also zero, $\dot{x}_G = 0$. This proves that as long as the assumption 2 holds, the nonholonomic WMR could only move in direction perpendicular to the wheels axis.

The definition of relationship between velocity of generalized coordinate vector $\dot{q}_G$ as an output and controlled input of linear υ and angular ω velocities is given by the following matrix:

$$\begin{bmatrix} \dot{x}_G \\ \dot{y}_G \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & l\sin\theta \\ \sin\theta & l\cos\theta \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \upsilon \\ \omega \end{bmatrix} = G(\theta) \begin{bmatrix} \upsilon \\ \omega \end{bmatrix}. \tag{11}$$

Through the application of equation (3) in (11), it is presented the relationship between the velocity of generalized coordinate vector $\dot{q}_G$ and the controlled angular velocities of right respectively left wheel:

$$\dot{q}_G = G(\theta) \cdot D \begin{bmatrix} \dot{\theta}_r \\ \dot{\theta}_l \end{bmatrix}. \tag{12}$$

Now equation (12) represents the kinematic model of the WMR in implicit form. The explicit form of a kinematic model of WMR is given by the equation (13).

$$\begin{bmatrix} \dot{x}_G \\ \dot{y}_G \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \frac{r}{2}\cos\theta - \frac{r}{2L} l\sin\theta & \frac{r}{2}\cos\theta + l\sin\theta \\ \frac{r}{2}\sin\theta + \frac{r}{2L} l\cos\theta & \frac{r}{2}\sin\theta - \frac{r}{2L} l\cos\theta \\ \frac{r}{2L} & \frac{r}{2L} \end{bmatrix} \begin{bmatrix} \dot{\theta}_G \\ \dot{\theta}_l \end{bmatrix} \tag{13}$$

Since at the output of the system the Degree of Freedom (DoF) which need to be controlled $[\dot{x}_G, \dot{y}_G, \dot{\theta}]$ is three, and at the input of the system the level of controllable DoF is two $[\dot{\theta}_r, \dot{\theta}_l]$, we confirm that the system (13) is nonholonomic. A system is nonholonomic when the controllable degree is less than the total degree which needs to be controlled, otherwise, the system is holonomic.

According to remarks (page 187) of [20], nonholonomic systems do not satisfy Brockett condition. Therefore, by using continuous control laws, it is impossible to arrive smooth asymptomatic stability at the desired point. However, approximated asymptotic stability region could be achieved, (see the last paragraphs of Section 4).

3. DYNAMIC MODELLING

Let's assume that both wheels will be rotated with the same angular velocity, but opposite direction of rotation, thus robot will rotate around its center of wheel axis (point B), as a result dynamic torque would act, and the point G will pass a circle with radius l. Therefore, robot on the way to the final position and orientation, will create a trajectory by moving within this circle, see Figure 3. When the robot gets a curved road, at the center of gravity acts resultant acceleration, which could be expressed as:

$$\vec{a}_R = \vec{a}_d + \vec{a}_r + \vec{a}_{cor} \qquad (14)$$

denoting by $\vec{a}_d$ – the displacement acceleration, $\vec{a}_r$ – the relative acceleration and $\vec{a}_{cor}$ – Coriolis acceleration. The displacement and relative acceleration can be separated into their normal and tangential components.

$$\vec{a}_R = \vec{a}_{dn} + \vec{a}_{dt} + \vec{a}_{rn} + \vec{a}_{rt} + \vec{a}_{cor} \qquad (15)$$

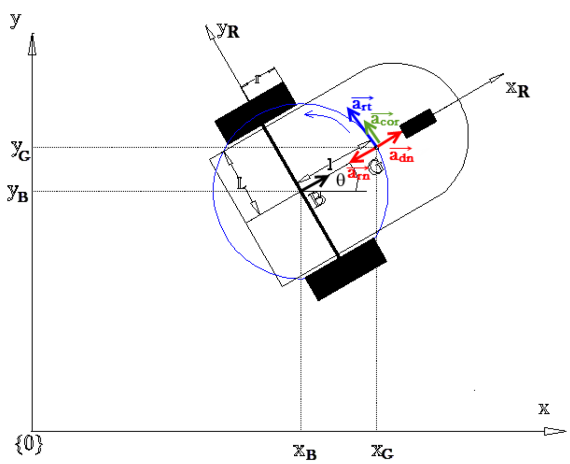

Fig. 3. Radial and tangential acceleration components

Since the WMR is nonholonomic it means that robot do not make displacement perpendicular to the wheel axis, hence $\vec{a}_{dt} = 0$. Considering that distance l doesn't change is constant, means that $\vec{a}_{dn}$ is same for point B and point G.

The acceleration of the center of gravity G could be found by the derivation of equation (8)

$$a_G = (\upsilon - l\dot{\theta}^2)e^{j\theta} + j(l\ddot{\theta} + \upsilon\dot{\theta})e^{j\theta}. \qquad (16)$$

The first component is the radial component, while the second component is the tangential component.

For the simplicity of understanding the equation (16), refer to the Figure 3. The correlation between (15) and (16) might be presented as:

$$a_G = (a_{dn} - a_{rn})e^{j\theta} + j(a_{rt} + a_{cor})e^{j\theta}. \qquad (17)$$

The forward movement is produced by the dynamic force F_d and the rotational motion is produced by the dynamic torque τ_d. The magnitude of these forces are given by the following equation:

$$\begin{aligned} F_d &= m \cdot a_{rad} = m \cdot (\dot{\upsilon} - l\dot{\theta}^2) \\ \tau_d &= (l_g + ml^2)\ddot{\theta} + ml\upsilon\dot{\theta} \end{aligned} \qquad (18)$$

where: m is the total mass of the platform without wheels, I_g is the moment of inertia calculated for rotation around the center of mass. The dynamic force F_d and dynamic torque of the robot τ_d are generated by the dynamic driven torque of the right τ_{mr} and left τ_{mi} motors:

$$\begin{aligned} F_d &= \frac{(\tau_{mr} + \tau_{ml})}{r} \\ \tau_d &= L\frac{(\tau_{mr} - \tau_{mr})}{r} \end{aligned} \qquad (19)$$

The dynamic model of WMR is represented in matrix form by merging the equations (18) and (19):

$$M\dot{\upsilon} + C(\upsilon) = B\tau \qquad (20)$$

where:

$$\left.\begin{aligned} M &= \begin{bmatrix} m & 0 \\ 0 & I_g + ml^2 \end{bmatrix}, \\ C(\upsilon) &= \begin{bmatrix} -ml\dot{\theta}^2 \\ ml\upsilon\dot{\theta} \end{bmatrix}, \\ B &= \begin{bmatrix} \frac{1}{r} & \frac{1}{r} \\ \frac{L}{r} & -\frac{L}{r} \end{bmatrix}, \end{aligned}\right\} \qquad (21)$$

The matrix $\boldsymbol{M}$ represents a positive definite inertial matrix, matrix $\boldsymbol{C}$ represents Coriolis and centrifugal matrix, $\boldsymbol{B}$ represents the input transformation matrix, $\tau = [\tau_{dr}, \tau_{dl}]^T$ and $\dot{\upsilon} = [\dot{\upsilon}, \dot{\omega}]^T$ represent vectors of controlled input dynamic torques and controlled output accelerations.

The dynamic model (20) is based on the coordinates of the WMR platform, for better modelling the physical system of WMR, the dynamic model should be extended including the dynamic models of actuating motors. The equation of motion could be written as:

$$\begin{aligned} I_{wm}\dot{\omega}_r + \tau_{dr} &= \tau_{mr} - \tau_{fr} \\ I_{wm}\dot{\omega}_l + \tau_{dl} &= \tau_{ml} - \tau_{fl} \end{aligned} \tag{22}$$

denoting by: I_{wm} – the inertia of each wheel plus the inertia of motor including the rotor inertia, τ_{mr}, τ_{ml} – the torque exerted from right, respectively left motor, and τ_{fr}, τ_{fl} – the friction torque from right respectively left motor.

The dynamic model of WMR including the dynamic of wheels plus motors could be defined through the substitution of equation (22) in (20) as:

$$M_{wm}\dot{\upsilon} + C(\upsilon) + B\tau_f = B\tau_m \tag{23}$$

where:

$$\left.\begin{aligned} M_{wm} &= \begin{bmatrix} m & \frac{2}{r^2} I_{wm} \\ 0 & I_g + ml^2 + \frac{2L^2}{r^2} I_{wm} \end{bmatrix}, \\ C(\upsilon) &= \begin{bmatrix} -ml\dot{\theta}^2 \\ ml\upsilon\dot{\theta} \end{bmatrix}, \\ B &= \begin{bmatrix} \frac{1}{r} & \frac{1}{r} \\ \frac{L}{r} & -\frac{L}{r} \end{bmatrix}, \\ \tau_m &= \begin{bmatrix} \tau_{mr} \\ \tau_{ml} \end{bmatrix}, \tau_f = \begin{bmatrix} \tau_{fr} \\ \tau_{fl} \end{bmatrix}, \end{aligned}\right\} \tag{24}$$

The matrix M_{wm} indicate the reduced form of positive definite Inertia matrix, while τ_m and τ_f represent vectors of generated motor torque and friction torque respectively. The dynamic modelling could be derived also by using Lagrange dynamic equation of motion. The dynamical modelling of two nonholonomic WMR using Lagrange formalism could be found in [21].

4. CONTROL STRATEGY

The control problem of robot stabilization could be separated in two individual control problems: robot positioning control and robot orientation control. The RPC must provide a control in such a way that robot will achieve the desired position (x_d, y_d), regardless the orientation of the robot. The ROC besides achieving the desired position must assure achieving desired orientation of the robot (x_d, y_d, θ_d).

The intention of control engineering is to find a feedback stabilizable controller, such that, the equilibrium point of the closed-loop system is asymptotically stable. Since the system is nonlinear and non-holonomic, it means that there do not exist smooth time invariant state feedback controller, which renders the equilibrium point of a closed loop system being asymptotically stable.

A) *Robot position control*

The control problem is to find a solution to bring the WMR to the final position regardless the orientation. Since the Cartesian coordinates of the actual position of the robot are known from the GPS sensor, and coordinates of the final position are known to us from the task request (x_d, y_d), then it is possible through simple equations to calculate the distance to a final position. The illustration of the problem is presented in Figure 4, denoted by Δr – the distance to the final position, α – the angle between the final position and Cartesian system, and θ – the heading angle of the robot.

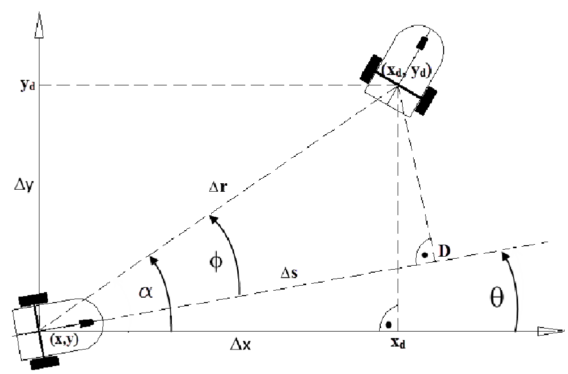

Fig. 4. Geometric solution of RPC

In order to solve the problem, assume a point D somewhere in the line of robot heading direction with distance Δs, such that, it will be the closest point from the final point. The angle to the final position from the heading orientation of the robot is defined as ϕ. In other words, it is the error between the assumed point D and the final point (desired position), defined as:

$$\cos\phi = \frac{\Delta s}{\Delta r} \tag{25}$$

The desired point (x_d, y_d) will be achieved if system control is designed, such that renders the $\Delta s \to 0$, $\phi \to 0$. Therefore, the positioning control problem will be solved by implementing such control strategy that provides such convergences.

The distance error respectively angular error are denoted by e_S, e_ϕ, and defined as:

$$e_s = \Delta s = \Delta r \cdot \cos\phi = \\ = \sqrt{(x_d - x)^2 + (y_d - y)^2} \cdot \cos\phi = \\ = \sqrt{(\Delta x)^2 + (\Delta y)^2} \cdot \cos\phi \quad (26)$$

$$e_\phi = \phi = \alpha - \theta = \arctan\left(\frac{y_d - y}{x_d - x}\right) - \theta = \\ = \arctan\left(\frac{\Delta y}{\Delta x}\right) = \theta \quad (27)$$

Through the implementation of above equations, in Figure 5, is presented the Block scheme of proposed control strategy for robot positioning.

The dynamic model is related with variable s and θ, by the following substitution

$$\upsilon = [\upsilon, \omega]^T = [\dot{s}, \dot{\theta}]^T .$$

The relation between the displacement of right and left wheel and the control signal s and θ, could be expressed by taking the inverse of equation (3), integrating it on both sides of expression and neglecting the integration constants.

$$\begin{bmatrix} \theta_r \\ \theta_l \end{bmatrix} = D^{-1} \begin{bmatrix} s \\ \theta \end{bmatrix} = D^{-1} \begin{bmatrix} u_s \\ u_\theta \end{bmatrix} = \begin{bmatrix} \frac{1}{r} & \frac{L}{r} \\ \frac{1}{r} & -\frac{L}{r} \end{bmatrix} \begin{bmatrix} u_s \\ u_\theta \end{bmatrix} \quad (28)$$

Equation (28) is used in the simulation environment to generate the reference inputs (outputs) on the wheels actuator control systems, when the vector $[s, \theta]^T$ is substituted by the control vector $[u_s, u_g]^T$.

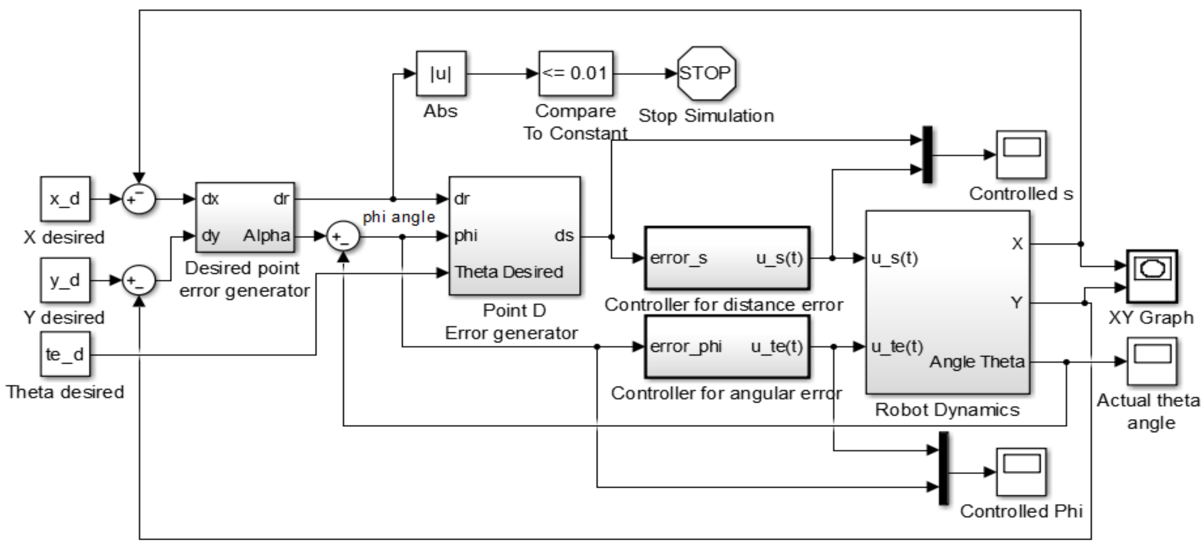

Fig. 5. Block schema of RPC Structure

B) *Robot orientation control*

The robot is not supposed to move straight to the final position, therefore, the control strategy design will take in consideration the orientation of the WMR at the final position.

The difference between the desired orientation angle θ_d and the angle to the final position α is defined by β, as $\beta = \theta_d - \alpha$. In order to solve the problem of the desired final orientation of the robot, assume a reference point R, as we would have rotated a final point for angle β in a clockwise direction, related to point B with radius Δr, see Figure 6.

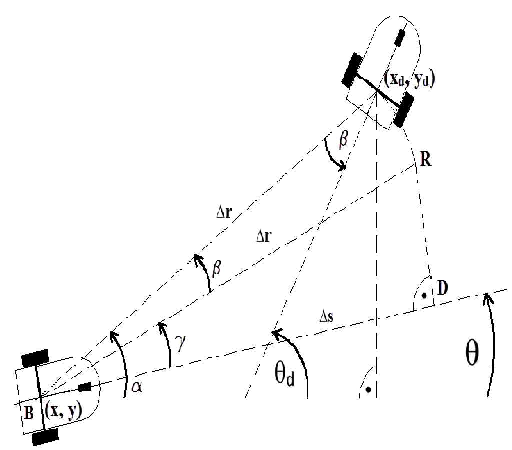

Fig. 6. Geometric solution of RPC and ROC

As the robot moves forward closer to the final posture, the desired orientation angle θ_d and final posture angle α will keep increasing, while β decreases. Continuously, as β decreases the reference point R will attempt to approach the final point with desired orientation.

Now, the angle between the heading direction of the robot and the reference point is denoted by γ, which could be expressed as

$$\gamma = e_\phi - \beta = 2\alpha - \theta_d - \theta.$$

The distance error respectively the angular error to the final orientation are denoted by e_s, e_θ as:

$$e_s = \Delta s = \Delta r \cdot \cos(\gamma) = \Delta r \cdot \cos(2\alpha - \theta_d - \theta)$$
$$e_\theta = \gamma$$

By applying the above formulas of this paper, in Figure 7 is presented the block scheme of proposed control strategy for position and orientation of WMR.

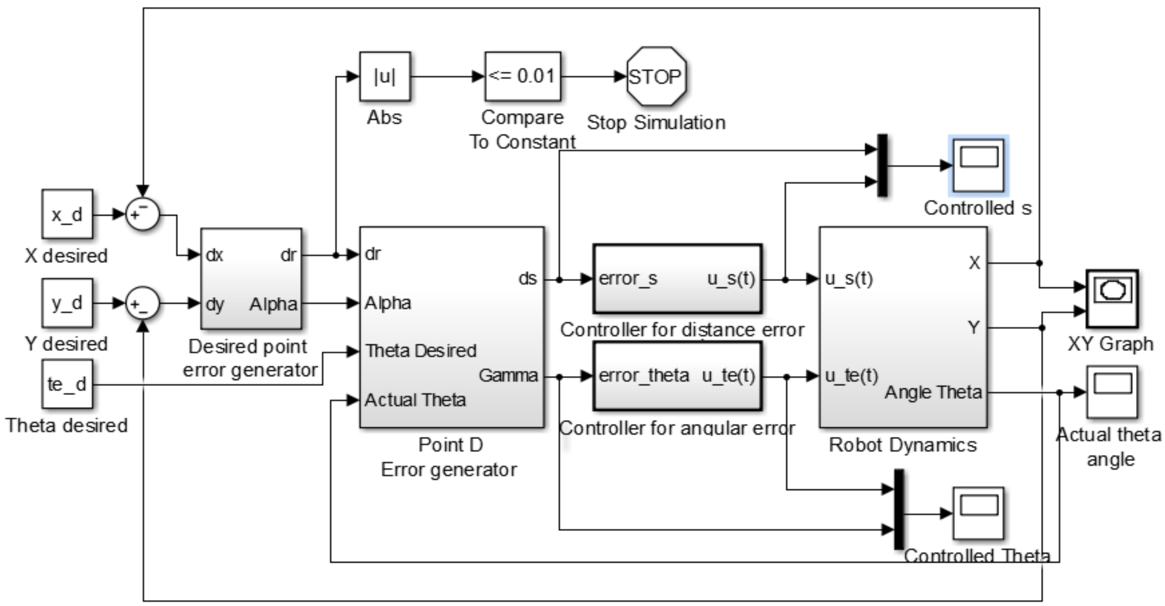

Fig. 7. Block scheme of RPC and ROC structures

In order to encapsulate the idea of robot stabilization about the desired posture, it could be summarized that: RPC attempts to move the robot toward point D, but simultaneously as $t \to \infty$. $\gamma \to 0$ point D approaches reference point R. While ROC as $t \to \infty$. $\beta \to 0$ will try to approach continuously reference point R toward final posture. The accuracy and sensitivity of sensor used for measuring the position of the platform, determines the circular region from the final point with a radius ε. When the mobile robot gets within this circular region $\Delta r \le \varepsilon$, then it is approximated that both linear error e_s and angular error e_θ are zero. Therefore, at this point the approximated asymptotic stability problem is accomplished based on the geometric approach, wherein the proposed control strategy is subjected too.

Often in text books, approximated asymptotic stability region is referred as asymptotic stability, so we do in this paper.

5. SIMULATION RESULTS

The simulation results are obtained by using Matlab/SIMULINK. Since it is considered slow-speed operating two-wheeled mobile robot, any linear controller could be used for the proposed control strategy for stabilization of the robot about a desired posture. In this paper, two PI controllers are used, one for controlling the distance error e_s and the other for controlling the angular error e_θ.

$$\left.\begin{aligned} u_s(t) &= K_{ps} \cdot e_s(t) + K_{is} \cdot \int_0^t e_s(t)dt, \\ u_\theta(t) &= K_{p\theta} \cdot e_\theta(t) + K_{i\theta} \cdot \int_0^t e_\theta(t)dt, \end{aligned}\right\} \qquad (29)$$

The performance of PI controllers could be adjust by tuning the gain parameters. Equation (23) is used to model Robot Dynamics block. The physical parameters taken to model the robot in the simulation are given: $m = 25$ kg, $r = 0.15$ m, $L = 0.3$ m, $l = 0.35$ m, $I_g = 0.25$ kg·m^2, $I_m = 0.01$ kg·m^2. The outer part and the heading shape of the virtual platform of a wheeled mobile robot is illustrated in Figure 8.

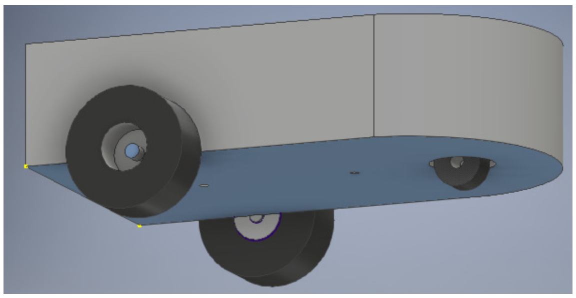

Fig. 8. Virtual platform of WMR

Various robot paths starting from the initial posture [0, 0, 0^o] toward the final point [20. 10] with different desired orientation angles θ_d are presented in Figure 9. Where paths with positive desired orientation angle $\theta_d > 0$ are presented with dash lines, while with solid lines are presented the negative ones. When the desired orientation angle is large, the robot needs to take a longer path.

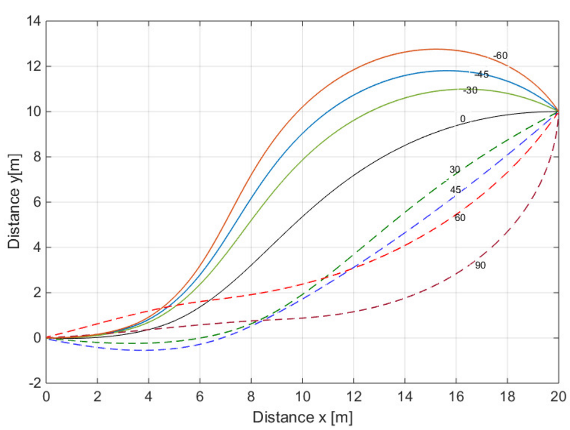

Fig. 9. Robot paths with various final desired orientation angle

In order to evaluate the efficiency of the proposed control strategy, in Figure 10 we have taken a scenario in such a way that robot starts in various initial points and goes to center point [0, 0]. For initial points, $x \geq 0^+$ their final desired orientation angle is taken 180^o, while for $x \leq 0^-$ the final desired orientation angle is taken 0^o.

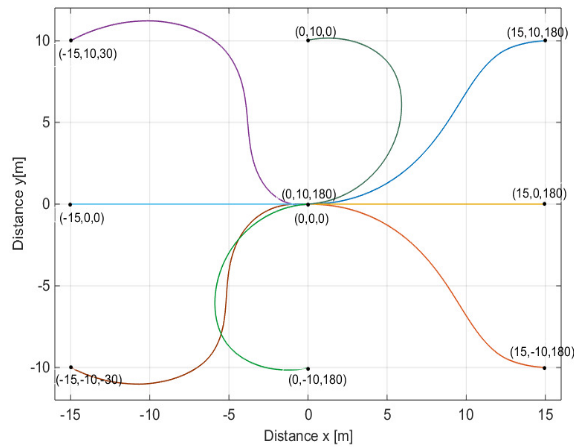

Fig. 10. Robot paths from various initial posture

In the following figures, according to a particular simulation of the robot path, starting from initial posture [0, 0, 0^o] and going toward final posture [20, 10, 0^o], are presented the convergences of angular and distance errors of RPC and ROC.

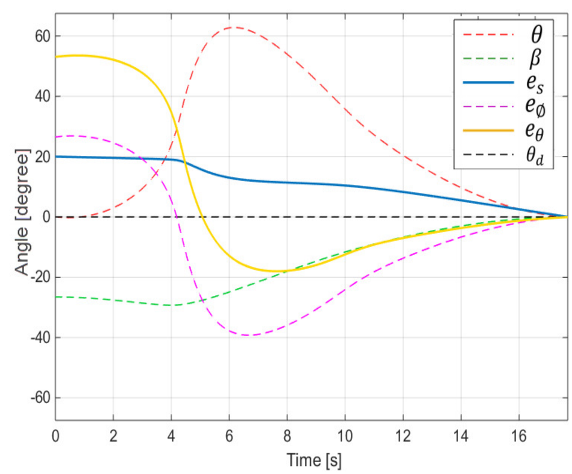

Fig. 11. Angular and distance errors of RPC and ROC for particular robot path

Furthermore, the angular velocity of a platform and the angular velocities of the left respectively right wheel are presented in Figure 12, but for the sake of a better illustration $\dot{\theta}_l$ and $\dot{\theta}_r$ are multiplied by a factor 0.1.

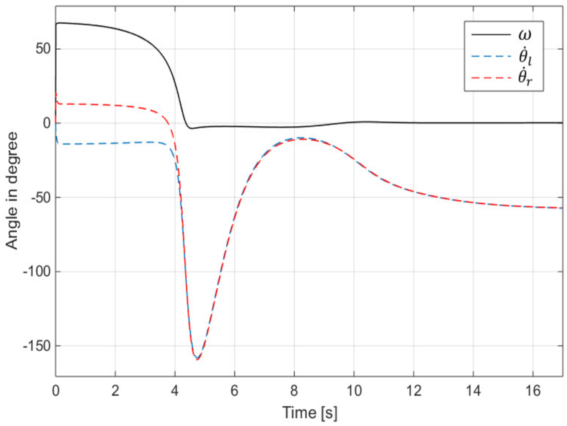

Fig. 12. Angular velocity and angular velocities of left and right wheel

The simulation results prove that an asymptotic stability could be achieved.

6. CONCLUSION

Even thought, this WMR system is nonlinear and non-holonomic, the proposed control strategy assures asymptotic stability about the desired posture. The simulation has shown very satisfactory results and proved that as $t \to \infty$. an asymptotic stability could be achieved. Furthermore, a robot needs to take longer path when the desired final orientation angle is large, this is not an advantage but still it is admissible for different practical applications.

The proposed control strategy is implementable and only requires localization of the robot, the performance of the WMR could be adjusted by tuning the gain parameters of PI controllers. It is applicable only for the configuration of two differential wheeled mobile robot.

Future works will be focused on implementing the proposed control strategy in the real WMR, compare and evaluate the performance in the real situation.

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