Journal of Electrical Engineering and Information Technologies, Vol. **7,** No. 2, pp. 87–96 (2022)
Article 199
Received: November 21, 2022
Accepted:December 5, 2022
In print: ISSN 2545–4250
On line: ISSN 2545–4269
UDC: 681.5.037.26:519.71
DOI: https://doi.org/10.51466/JEEIT2271199087d

Original scientific paper

Dedicated to late Professor Milčo Kocare, the great teacher of Electronics, Communications and Automation, while honoring our colleagues Goce Arsov and Ljupčo Panovski and the hospitality of FEIT Institutes of Electronics and of Automation & Systems Engineering

STATE FEEDBACK H_∞ CONTROL FOR A CLASS OF SWITCHED FUZZY SYSTEMS

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A b s t r a c t: An innovated model of the switched fuzzy systems whose subsystems are fuzzy systems is presented. State feedback H_∞ control for this class of fuzzy systems is studied using theory of switching systems and control by employing single Lyapunov function technique. A switching strategy of the switched fuzzy system with continuous-time control input and a relevant switching law is developed. The main condition for asymptotic stability of the equilibrium state is given in form of convex combinations of linear matrix inequalities, which are solvable by LMI Toolbox and Matlab-Simulink platform. Application to a room regulated air-conditioning plant and the respective simulation results are presented to demonstrate the effectiveness and feasible operating performance of the hybrid control design.

Key words;; switching control; fuzzy logic control; switched fuzzy systems;, state feedback H_∞ control

СОСТОЈБЕНО H_∞ УПРАВУВАЊЕ ПО ПОВРАТНА ВРСКА ЗА КЛАСА НА ПРЕВКЛУЧУВАЧКИ ФАЗИ-СИСТЕМИ

А п с т р а к т: Трудот презентира иновиран модел на превклучувачки фази-системи чии потсистеми се исто така фази-системи. Проучувано е состојбено H_∞ управување по повратна врска за оваа класа на системи со користење на теоријата на превклучувачки системи и на техниката на единична функција на Љапунов. Развиени се превклучувачка стратегија за превклучувачкиот фази-систем со континуиран управувачки влез и соодветен закон за превклучување. Главниот услов за асимптотска стабилност на рамнотежната состојба е даден во облик на конвексни комбинации на линеарни матрични неравенства кои се решаваат со LMI Toolbox во платформата Matlab-Simulink. Презентирана е апликација на ваквиот пристап при управување на процес за кондиционирање на воздух во затворена просторија, како и соодветни симулациски резултати. Со тоа се демонстрираат остварливоста и ефикасноста на изложениот хибриден дизајн на управување.

Клучни зборови: превклучувачко управување; фази логичко управување; превклучувачки фази-системи; состојбено H_∞ управување по повратна врска

1. INTRODUCTION

In recent years, considerable attention has been paid to analysis and synthesis of switched systems [1–4]. Switched systems represent one important class of hybrid systems [5, 22, 23]. A switched system consists of a number of sub-systems, either continuous- or discrete-time dynamic systems, along with a relevant switching law that orchestrates the switching between its sub-systems.

Important applications such as in computer disc drives [5], some robot control [6], cart-pendulum systems [7], and recent aero-space developments emphasized switched systems have extensive engineering background in practice in particular [18, 19]. Their theoretical significance and practical value paved a flourishing trend to study switched systems.

On the other hand, fuzzy logical control [8, 24, 25] has emerged as one of the most active and fruitful areas. In the recent past, certain rather useful techniques for stability analysis and synthesis have emerged due to the methodology of Linear Matrix Inequalities (LMI), its scientific background and its computing technology. The LMI-based designs for Takagi-Sugeno (T-S) fuzzy systems have sparked a trend toward the fuzzy control theory and design techniques [8, 20–22]. The LMI techniques are employed to solve an H_∞ control problem of a nonlinear control system via robust H_∞ fuzzy control [9]. A thorough study of stability analysis and synthesis of nonlinear time-delay systems via linear T-S fuzzy models by state feedback, includeng stabilization of uncertain fuzzy systems, has been explored in [10, 11] by using the LMI techniques. In particular, the H_∞ control problem for uncertain discrete-time fuzzy systems by state feedback has been considered in [11]. In [12], the mixed H_2/H_∞ fuzzy feedback control problems using LMIs have been considered [17, 19, 25, 26], which are further developed and extended in recent studies [27, 28, 30].

A switched system is called a switched fuzzy system if all subsystems are fuzzy systems. This class of systems can often more precisely describe continuous dynamics and discrete dynamics as well as their interactions in actual systems. Compared with the results on stability of switched systems and those of fuzzy control systems, the results on switched fuzzy systems are very few. In [13], the combination of hybrid systems and fuzzy multiple model systems is described, and a fuzzy switched hybrid controller is put forward. In [14, 15], a switching fuzzy model is studied and stability conditions are given as well as [16–18] give some extension based on [14, 15]. Such a switching fuzzy system model has two levels of structure, which the first level is region rule level and the second level is a local fuzzy rule level. This model is switching in local fuzzy rule level of the second level according to the premise variable in region rule level of the first level, which promise wide applications [29, 32, 33].

An innovated model for a class of switched fuzzy systems and its fuzzy-logic based control is proposed in this paper, which differs from existing ones. It represents essentially a switched system whose sub-systems all are fuzzy systems. The respective synthesis design methods do inherit some features of hybrid systems, but involves information flow of fuzzy systems. The state feedback H_∞ robust control is investigated exploiting the idea that control infrastructure too should be derived employing a similar fuzzy-rule model to that of the plant system. In contrast to many existing results, in here studied switched fuzzy system control is rather relying on the intuitive *T-S* fuzzy-rule models.

This approach provides a kind of different premise variable switching directly, while works in aforementioned [14–18] considered a model with two-level structure. Synthesis design of both continuous-time controllers for subsystems and switching law has been developed. Furthermore, based on single Lyapunov function technique, a sufficient condition for the switched fuzzy systems to be asymptotically stable with H_∞-norm bound is derived. Finally, by using Matlable's Fuzzy Toolbox, LMI Toolbox and Simulink, the obtained simulation results for the application to for room air-conditioning plant, on its regulating system, demonstrate the effectiveness and feasible performance of this novel control design synthesis. References follow thereafter.

2. SYSTEM MODEL AND PRELIMINARIES

Consider the continuous-time uncertain switched fuzzy model of Takagi-Sugeno class. In this class of *T-S* switched fuzzy systems every subsystem systems is an uncertain fuzzy system as follows:

$$\begin{aligned} &R^l_{\sigma(t)}: \text{if } \xi_1 \text{ is } M^l_{\sigma(t)1} \cdots \text{and } \xi_p \text{ is } M^l_{\sigma(t)p}, \text{ then} \\ &\quad \dot{x}(t) = A_{\sigma(t)l}x(t) + B_{1\sigma(t)l}w_{\sigma(t)}(t) + B_{2\sigma(t)l}u_{\sigma(t)}(t), \\ &\quad z(t) = C_{\sigma(t)l}x(t) + D_{\sigma(t)l}u(t), \quad l = 1,2,\cdots N_{\sigma(t)}. \end{aligned} \tag{1}$$

Quantities in (1) denote: $\xi_1, \xi_2, \cdots, \xi_p$ are the fuzzy-set premise variables;

$$\sigma(t): R_+ \to M = \{1,2,\cdots,m\}$$

is a piecewise constant function, called a switching sequence signal; $M^l_{\sigma1}, \cdots, M^l_{\sigma p}$ denote fuzzy sets in the σ-th switched subsystem; $R^l_{\sigma(t)}$ denotes the l-th fuzzy inference rule in the σ-th switched subsystem; $N_{\sigma(t)}$ is the number of inference rules in the σ-th switched subsystem such that fuzzy rules are selected in every switched subsystem; $u_{\sigma(t)}(t)$ is the control input of the σ-th switched subsystem; $x(t)$ is the system state variable vector, $z(t)$ is the output to be controlled, while $w_{\sigma(t)}(t)$ is disturbance input of the σ-th switched subsystem; matrices $A_{\sigma(t)l}$, $B_{1\sigma(t)l}$, $B_{2\sigma(t)l}$ and $C_{\sigma(t)l}$, $D_{\sigma(t)l}$ are known constant matrices of appropriate dimensions of the σ-th switched subsystem.

It should be noted further that the i-th switched subsystem appears in the form:

$$R_i^l: \text{ if } \xi_1 \text{ is } M_{i1}^l \cdots \text{and } \xi_p \text{ is } M_{ip}^l, \text{ then}$$

$$\begin{aligned} &\dot{x}(t) = A_{il}x(t) + B_{1il}w_i(t) + B_{2il}u_i(t), \\ &z(t) = C_{il}x(t) + D_{il}u(t), \\ &\quad l = 1, 2, \cdots N_i, \qquad i = 1, 2, \cdots, m. \end{aligned} \tag{2}$$

Then global or overall model of the i-th switched subsystem via *Zadeh's fuzzy-logic inference* [24–25] is described by:

$$\begin{aligned} &\dot{x}(t) = \sum_{l=1}^{N_i} \eta_{il}(\xi(t))\left[A_{il}x(t) + B_{1il}w_i(t) + B_{2il}u_i(t)\right], \\ &z(t) = \sum_{l=1}^{N_i} \eta_{il}(\xi(t))\left[C_{il}x(t) + D_{il}u_i(t)\right], \\ &\quad i = 1, 2, \cdots, m, \end{aligned} \tag{3}$$

where:

$$0 \le \eta_{il}\big(\xi(t)\big) \le 1, \quad \sum_{l=1}^{N_i} \eta_{il}\big(\xi(t)\big) = 1 \quad . \tag{4a}$$

$$w_{il}\left(\xi(t)\right) = \prod_{\rho=1}^{p} M_{i\rho}^l\left(\xi_\rho(t)\right), \tag{4b}$$

$$\eta_{il}\left(\xi(t)\right) = [w_{il}\left(\xi(t)\right)] \Big/ [\sum_{l=1}^{N_i} w_{il}\left(\xi(t)\right)]. \tag{4c}$$

Notice, in here, quantity $M_{i\rho}^l\big(\xi_\rho(t)\big)$ denotes the fuzzy-set membership function and $\xi_\rho(t)$ belon gs to the fuzzy set $M_{i\rho}^l$.

Now, H_∞ control problem for the switched fuzzy system (1) can be stated as follows:

Let a constant $\gamma > 0$ be given. Find a continuous-time state-feedback controller $u_i = u_i(x)$ for each sub-system, and a relevant switching law $i = \sigma(t)$ such that:

(1) The closed-loop control system is asymptotically stable whenever $w_i = 0$.

(2) The output z satisfies $\|z\|_2 \le \gamma \|w_i\|_2$ beginning with zero initial condition, which is typical driving mode to operating steady-state equilibrium.

Fuzzy systems partition the state space into many fuzzy sub-areas, and local model is designed in every fuzzy sub area. Global model of fuzzy system is composed of a series of local model which is linked by fuzzy membership function. If all the sub-systems of the considered switched system are all *T-S* fuzzy systems (or any other class of fuzzy system models for that matter), then such systems represent the class of switched *T-S* fuzzy systems.

In fact, a sketch map of the switched fuzzy systems is depicted in Figure 1 where Ω_i denotes the system state area of the i-th switched subsystem. Ω_{il} denotes the l-th fuzzy sub-area in Ω_i. It should be noted again, the switched fuzzy system partitions the Ω_i sub-area into l fuzzy sub-areas $\Omega_{i1}, \cdots, \Omega_{il}, \cdots, \Omega_{i\ell}$. There is local linear model in every fuzzy sub-area, namely local linear model in Ω_{il} is its state-space model,

$$\dot{x}(t) = A_{il}x(t) + B_{il}u_i(t).$$

The model of every switched sub-area $\Omega_1, \cdots, \Omega_i, \cdots, \Omega_m$ is composed of local linear models which are linked by the fuzzy set membership functions.

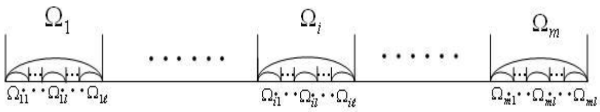

Fig. 1. A descriptive map of switched fuzzy systems in their state space

The design of the switching law for fuzzy sub-area model is carried out so as to ensure stability of the overall switched fuzzy system. When local model in fuzzy sub-area satisfies the switching law, then the switch goes to the Ω_i-th sub-system to ensure stability of the switched fuzzy system.

3. MAIN NOVEL RESULTS

This section derives a condition for the H_∞ control problem to be solvable and presents design synthesis employing continuous-time controllers for subsystems and a switching law. Here, the methodology due to Tanaka and coauthors [14–16] for PDC fuzzy controller design is being used for every fuzzy sub-system [8].

Namely, as the plant system (2) also the fuzzy controller together both are assumed to have the same fuzzy inference premise variables. Therefore, in terms of Takagi-Sugeno fuzzy-rule models, it follows:

$$R_{ic}^l: \text{ if } \xi_1 \text{ is } M_{i1}^l \cdots \text{ and } \xi_p \text{ is } M_{ip}^l, \text{ then}$$
$$u_i(t) = K_{il}x(t),\ l = 1,2,\cdots N_i,\ i = 1,2,\cdots,m. \tag{5a}$$

Thus again following Zadeh's fuzzy logic inference [14–25], globally the overall control is inferred as follows:

$$u_i(t) = \sum_{l=1}^{N_i}\eta_{il}K_{il}x(t)\ ,\ i = 1,2,\cdots,m. \tag{5b}$$

Then globally the overall model of the i-th fuzzy sub-system is described by:

$$\begin{aligned}\dot{x}(t) &= \sum_{l=1}^{N_i}\eta_{il}\sum_{r=1}^{N_i}\eta_{ir}\left[A_{il}x(t) + B_{1il}w_i + B_{2il}K_{ir}x(t)\right]\\ z(t) &= \sum_{l=1}^{N_i}\eta_{il}\sum_{r=1}^{N_i}\eta_{ir}\left(C_{il} + D_{il}K_{ir}\right)x(t)\end{aligned} \tag{6}$$

Lemma 1. Let $a_{ij_i}\ (1 \le i \le m, 1 \le j_i \le N_i)$ be a set of constants satisfying

$$\sum_{i=1}^{m}a_{ij_i} < 0,\ \forall 1 \le j_i \le N_i\ .$$

Then, there exists at least one i such that

$$a_{ij_i} < 0,\ 1 \le j_i \le N_i\,.$$

Proof. It is trivial hence omitted.

Theorem 1. Let a constant $\gamma > 0$ be given. Suppose there exist a positive definite matrix P and constant $\lambda_{ij_i} > 0\ \ (i = 1,2\cdots m,\ j_i = 1,2\cdots N_i)$ such that

$$\sum_{i=1}^{m}\lambda_{ij_i}\begin{bmatrix}(A_{ij_i} + B_{2ij_i}K_{i\vartheta_i})^T P + P(A_{ij_i} + B_{2ij_i}K_{i\vartheta_i}) + \\ \dfrac{1}{\gamma^2}PB_{1ij_i}B_{1i\vartheta_i}^T P + (C_{ij_i} + D_{ij_i}K_{i\vartheta_i})^T \bullet \\ (C_{ip_i} + D_{ip_i}K_{iq_i})\end{bmatrix} < 0$$
$$i = 1,2,\cdots,m,\ \ j_i, \vartheta_i, p_i, q_i = 1,2,\cdots,N_i. \tag{7}$$

Then the state feedback controllers (5) and the following switching law (8) solve the investigated H_∞ control problem:

$$\sigma(x) = \arg\min\{\bar{V}_i(x) \overset{\Delta}{=} \max_{j_i,\vartheta_i,p_i,q_i}\{$$
$$x^T\begin{bmatrix}(A_{ij_i} + B_{2ij_i}K_{i\vartheta_i})^T P + P(A_{ij_i} + B_{2ij_i}K_{i\vartheta_i}) + \\ \dfrac{1}{\gamma^2}PB_{1ij_i}B_{1i\vartheta_i}^T P + (C_{ij_i} + D_{ij_i}K_{i\vartheta_i})^T \bullet \\ (C_{ip_i} + D_{ip_i}K_{iq_i})\end{bmatrix}x < 0,$$
$$j_i, \vartheta_i, p_i, q_i = 1,2,\cdots,N_i\}\}. \tag{8}$$

Proof. From (7) we know that for any $x \ne 0$, it holds true

$$\sum_{i=1}^{m}\lambda_{ij_i}x^T\begin{bmatrix}(A_{ij_i} + B_{2ij_i}K_{i\vartheta_i})^T P + P(A_{ij_i} + B_{2ij_i}K_{i\vartheta_i}) + \\ \dfrac{1}{\gamma^2}PB_{1ij_i}B_{1i\vartheta_i}^T P + (C_{ij_i} + D_{ij_i}K_{i\vartheta_i})^T \bullet \\ (C_{ip_i} + D_{ip_i}K_{iq_i})\end{bmatrix}x < 0,$$
$$i = 1,2,\cdots,m,\quad j_i, \vartheta_i, p_i, q_i = 1,2,\cdots,N_i. \tag{9}$$

Notice that (9) holds for any

$j_i, \vartheta_i, p_i, q_i \in \{1,2,\cdots N_i\}$ and $\lambda_{ij_i} > 0$.

The Lemma 1 guarantees that there exists at least an i such that for any $j_i, \vartheta_i, p_i, q_i$

$$x^T\begin{bmatrix}(A_{ij_i} + B_{2ij_i}K_{i\vartheta_i})^T P + P(A_{ij_i} + B_{2ij_i}K_{i\vartheta_i}) + \\ \dfrac{1}{\gamma^2}PB_{1ij_i}B_{1i\vartheta_i}^T P + (C_{ij_i} + D_{ij_i}K_{i\vartheta_i})^T \bullet \\ (C_{ip_i} + D_{ip_i}K_{iq_i})\end{bmatrix}x < 0 \tag{10}$$

Thus, the switching law defined by (10) is well-defined.

Next, as in [20, 23], let now calculate the time derivative of Lyapunov candidate function $V(x(t)) = x^T(t)Px(t)$:

$$\dot V = \dot x^T Px + x^T P\dot x =$$
$$= \sum_{l=1}^{N_i}\eta_{il}\sum_{r=1}^{N_i}\eta_{ir}\left(A_{il}x + B_{1il}w_i + B_{2il}K_{ir}x\right)^T Px +$$
$$+\sum_{l=1}^{N_i}\eta_{il}\sum_{r=1}^{N_i}\eta_{ir}x^T P\left(A_{il}x + B_{1il}w_i + B_{2il}K_{ir}x\right) =$$
$$= \sum_{l=1}^{N_i}\eta_{il}\sum_{r=1}^{N_i}\eta_{ir}x^T\begin{bmatrix}\left(A_{il}+B_{2il}K_{ir}\right)^T P + P\left(A_{il}+B_{2il}K_{ir}\right)+\\ \frac{1}{\gamma^2}PB_{1il}B_{1ir}^T P + \sum_{s=1}^{N_i}\eta_{is}\sum_{d=1}^{N_i}\eta_{id}\\ \left(C_{il}+D_{il}K_{ir}\right)^T\left(C_{is}+D_{is}K_{id}\right)\end{bmatrix}x +$$
$$+\sum_{l=1}^{N_i}\eta_{il}\left(w_i^T B_{1il}^T Px + x^T PB_{1il}w_i\right) -$$
$$-\sum_{l=1}^{N_i}\eta_{il}\sum_{r=1}^{N_i}\eta_{ir}x^T\begin{bmatrix}\frac{1}{\gamma^2}PB_{1il}B_{1ir}^T P + \sum_{s=1}^{N_i}\eta_{is}\sum_{d=1}^{N_i}\eta_{id}\\ \left(C_{il}+D_{il}K_{ir}\right)^T\left(C_{is}+D_{is}K_{id}\right)\end{bmatrix}x \tag{11}$$

The second term on the right-hand side of (11) is found:

$$\sum_{l=1}^{N_i}\eta_{il}\left(w_i^T B_{1il}^T Px + x^T PB_{1il}w_i\right) =$$
$$w_i^T\left(\sum_{l=1}^{N_i}\eta_{il}B_{1il}^T Px\right) + \left(\sum_{l=1}^{N_i}\eta_{il}B_{1il}^T Px\right)^T w_i \tag{12}$$

The last term on the right-hand of (11) is found:

$$\sum_{l=1}^{N_i}\eta_{il}\sum_{r=1}^{N_i}\eta_{ir}x^T\begin{bmatrix}\frac{1}{\gamma^2}PB_{1il}B_{1ir}^T P + \sum_{s=1}^{N_i}\eta_{is}\sum_{d=1}^{N_i}\eta_{id}\\ \left(C_{il}+D_{il}K_{ir}\right)^T\left(C_{is}+D_{is}K_{id}\right)\end{bmatrix}x =$$
$$\left(\frac{1}{\gamma}\sum_{l=1}^{N_i}\eta_{il}B_{1il}^T Px\right)^T\left(\frac{1}{\gamma}\sum_{r=1}^{N_i}\eta_{ir}B_{1ir}^T Px\right) +$$
$$\left[\sum_{l=1}^{N_i}\eta_{il}\sum_{r=1}^{N_i}\eta_{ir}\left(C_{il}+D_{il}K_{ir}\right)x\right]^T\left[\sum_{l=1}^{N_i}\eta_{il}\sum_{r=1}^{N_i}\eta_{ir}\left(C_{il}+D_{il}K_{ir}\right)x\right] \tag{13}$$

By virtue of (10), it follows:

$$x^T\left[(A_{ij_i}+B_{2ij_i}K_{i\vartheta_i})^T P + P(A_{ij_i}+B_{2ij_i}K_{i\vartheta_i})\right]x \le$$
$$x^T\begin{bmatrix}(A_{ij_i}+B_{2ij_i}K_{i\vartheta_i})^T P + P(A_{ij_i}+B_{2ij_i}K_{i\vartheta_i})+\\ \frac{1}{\gamma^2}PB_{1ij_i}B_{1i\vartheta_i}^T P + (C_{ij_i}+D_{ij_i}K_{i\vartheta_i})^T \bullet\\ (C_{ip_i}+D_{ip_i}K_{iq_i})\end{bmatrix}x < 0$$
$$j_i, \vartheta_i, p_i, q_i = 1, 2, \cdots, N_i. \tag{14}$$

When $w_i = 0$, by virtue of relationships (4 a, b, c), due to (11) and (14), one can calculate:

$$\dot V = \sum_{l=1}^{N_i}\eta_{il}\sum_{r=1}^{N_i}\eta_{ir}x^T\left[\left(A_{il}+B_{2il}K_{ir}\right)^T P + P\left(A_{il}+B_{2il}K_{ir}\right)\right]x < 0. \tag{15}$$

Thus, the system (1) in the closed loop and under controls (5) is asymptotically stable.

Combing (11), (12) and (13) gives rise to

$$\dot V \le \sum_{l=1}^{N_i}\eta_{il}\sum_{r=1}^{N_i}\eta_{ir}\sum_{s=1}^{N_i}\eta_{is}\sum_{d=1}^{N_i}\eta_{id}x^T Q_{ilrsd}x - z^T z + \gamma^2 w_i^T w_i -$$
$$\left(\gamma w_i - \frac{1}{\gamma}\sum_{l=1}^{N_i}\eta_{il}B_{1il}^T Px\right)^T\left(\gamma w_i - \frac{1}{\gamma}\sum_{l=1}^{N_i}\eta_{il}B_{1il}^T Px\right) \tag{16}$$

where

$$Q_{ilrsd} = \left(A_{il}+B_{2il}K_{ir}\right)^T P + P\left(A_{il}+B_{2il}K_{ir}\right)+$$
$$\frac{1}{\gamma^2}PB_{1il}B_{1ir}^T P + \left(C_{il}+D_{il}K_{ir}\right)^T\left(C_{is}+D_{is}K_{id}\right)$$

Without loss of generality, let suppose zero initial state $x(0) = 0$ and Lyapunov function value $V(x(0)) = 0$ at the initial state [20, 23]. Now, by re-arranging (16) and then solving the integral in it for t from 0 to ∞, one can calculate the following inequality yield:

$$\|z(t)\|_2^2 \le \|z(t)\|_2^2 - \lambda_{\max}\left(\sum_{l=1}^{N_i}\eta_{il}\sum_{r=1}^{N_i}\eta_{ir}\sum_{s=1}^{N_i}\eta_{is}\sum_{d=1}^{N_i}\eta_{id}Q_{ilrsd}\right)\|x(t)\|_2^2 +$$
$$+V(\infty) + \gamma^2\left\|w_i(t) - \frac{1}{\gamma^2}\sum_{l=1}^{N_i}\eta_{il}B_{1il}^T Px\right\|_2^2 \le \gamma^2\|w_i(t)\|_2^2$$

where

$$\lambda_{\max}\left(\sum_{l=1}^{N_i}\eta_{il}\sum_{r=1}^{N_i}\eta_{ir}\sum_{s=1}^{N_i}\eta_{is}\sum_{d=1}^{N_i}\eta_{id}Q_{ilrsd}\right)$$

denotes the maximal eigenvalue of matrix

$$\sum_{l=1}^{N_i}\eta_{il}\sum_{r=1}^{N_i}\eta_{ir}\sum_{s=1}^{N_i}\eta_{is}\sum_{d=1}^{N_i}\eta_{id}Q_{ilrsd}.$$

In addition, it should be noted that in the above derivations to establish results confirming system stability analysis [20] the well-known Schur Complement Lemma and Uncertainty Representation Lemma for specific symmetric matrices, in from literature [21–22] play important role of crucial tools. Both these lemmas are recalled below in here.

Lemma 2. (Schur Complement). For a given the symmetric

$$M=\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22}\end{bmatrix},$$

where M_{11} and M_{21} are symmetric matrices, the following inequality condition statements are equivalent:

1/. $M<0$,

2/. $M_{11}<0,\ M_{22}-M_{21}M_{11}^{-1}M_{12}<0$,

3/. $M_{22}<0,\ M_{11}-M_{12}M_{22}^{-1}M_{21}<0$.

Lemma 3. Assume the uncertainty $F(t)$ and matrices L, $M=M^T$, S, and N of appropriate dimensions. Then the following two condition statements are equivalent:

1/. $M+SF(t)N+S^TF^T(t)N^T<0$,

2/. For $\rho>0$ an existing deterministic or stochastic real-number it holds true

$$M=\begin{bmatrix} M & \rho S & N^T \\ \rho S^T & -\rho I & \rho L^T \\ N & \rho L & -\rho I\end{bmatrix}<0.$$

4. ILLUSTRATIVE EXAMPLE

In order to illustrate the just presented design analysis approach let consider its application to the stability control of problem of a room air regulating system [8, 19]. The state equation of the plant system is given as follows:

$$\ddot{T}_n=-(\frac{1}{T_1}+\frac{1}{T_2})\dot{T}_n-\frac{1}{T_1T_2}T_n+\frac{k_1k_2}{T_1T_2}u$$

In here quantities denote: T_n is the air temperature variable in air-conditioned room [°C]; $\dot{T}_n$ is the rate of air temperature variable of the air-conditioned room [°C/min]; T_1 is the empiric inertia time constant of the air-conditioned room [min]; k_1 is the amplifying coefficient (gain) of the room equilibrium constant temperature [℃/℃]; T_2 is the empiric inertia time constant of the steam heater [min]; k_2 is the gain coefficient of the electric heating actuator [°C/°C]; and u is the control input variable in terms of electrical power. The reported empirically found time constants are as follows:

*When room temperature is considered lower than human comfort sensing, $T_1=20.30$ min, $T_2=1$ min.

**When room temperature is considered higher than human comfort sensing, $T_1=30.40$ min, $T_2=2.5$ min.

In order to illustrate the stability control design analysis of this system, coordinate transformation is carried out so as to transform the problem into zero-state stability control. Taking into consideration the available redundancy of circuit actuator the common sense fuzzy model is converted into the switched fuzzy model to advance arriving at scheduled temperature rise speed of air regulating system operation. Therefore the dynamics of the considered air regulating system operation is approximate by the following *T-S* fuzzy rule based model:

R_1^1: if x_1 is P_{11}^1, *close to positive,* then

$$\dot{x}=A_{11}x+B_{111}w_1+B_{211}u_1,\ z=C_{11}x+D_{11}u_1,$$

R_1^2 : if x_1 is N_{11}^2, *close to negative,* then

$$\dot{x}=A_{12}x+B_{112}w_1+B_{212}u_1,\ z=C_{12}x+D_{12}u_1,$$

R_2^1: if x_1 is P_{21}^1 ,*close to positive,* then

$$\dot{x}(t)=A_{21}x+B_{121}w_2+B_{221}u_2,\ z=C_{21}x+D_{21}u_2,$$

R_2^2: if x_1 is N_{21}^2, *close to negative,* then

$$\dot{x}(t)=A_{22}x+B_{122}w_2+B_{222}u_2,\ z=C_{22}x+D_{22}u_2,$$

where:

$$A_{11}=\begin{bmatrix} -0.5 & 4 \\ -0.943 & -1.0493\end{bmatrix},$$

$$A_{12}=\begin{bmatrix}-0.5 & 3\\ -0.132 & -0.4529\end{bmatrix}$$

$$B_{111}=B_{112}=\begin{bmatrix}0\\1\end{bmatrix},\ B_{211}=\begin{bmatrix}0\\0.4926\end{bmatrix},$$

$$B_{212}=\begin{bmatrix}0\\0.1316\end{bmatrix},$$

$$A_{21}=\begin{bmatrix}1 & 2\\ -0.2941 & -1.4321\end{bmatrix},$$

$$A_{21}=\begin{bmatrix}1 & 2\\ -0.4706 & -0.7535\end{bmatrix},$$

$$B_{121}=B_{122}=\begin{bmatrix}0\\1\end{bmatrix},\ B_{221}=\begin{bmatrix}0\\0.5765\end{bmatrix},$$

$$B_{222}=\begin{bmatrix}0\\0.1765\end{bmatrix},$$

$$C_{11}=C_{12}=\begin{bmatrix}1 & 1\end{bmatrix},\ C_{21}=C_{22}=\begin{bmatrix}1 & 0\end{bmatrix},$$

$$D_{11}=D_{12}=0.03,\ D_{21}=D_{22}=0.04\,.$$

The issue of defining appropriate membership functions as pointed out by Lotfi A. Zadeh [25], the founder and inventor of fuzzy logic and fuzzy systems [25], is essentially an application dependent problem. Because it is dependent on the universe of discourse set (i.e. physical nature of plan's state space) hence it is open to exploration within the context of the given case study application. This issue was thoroughly explored in the studies [8, 19], and concluded that fuzzy-set membership functions defined below have wide applicability and usually are optimum statedependent ones:

$$\mu_{P_{11}^1}(x_1)=\mu_{P_{21}^1}(x_1)=1-\frac{1}{1+e^{-2x_1}},$$

$$\mu_{N_{11}^2}(x_1)=\mu_{N_{21}^2}(x_1)=\frac{1}{1+e^{-2x_1}}.$$

Further, let explore the considerable case with robustness index level $\gamma=1$. Then, due to the derived state-feedback control formula:

$$u_i(t)=\sum_{l=1}^{N_i}\eta_{il}K_{il}x(t),\ i=1,2,$$

the inequality of Theorem 1 is subject to evaluating calculation of the following matrix inequality:

$$\sum_{i=1}^{2}\lambda_{ij_i}\begin{bmatrix}(A_{ij_i}+B_{2ij_i}K_{i\vartheta_i})^T P+P(A_{ij_i}+B_{2ij_i}K_{i\vartheta_i})+\\ \frac{1}{\gamma^2}PB_{1ij_i}B_{1i\vartheta_i}^T P+(C_{ij_i}+D_{ij_i}K_{i\vartheta_i})^T\bullet\\ (C_{ip_i}+D_{ip_i}K_{iq_i})\end{bmatrix}<0$$

$$i=1,2,\quad j_i,\vartheta_i,p_i,q_i=1,2,\cdots,N_i. \tag{17}$$

Without loss of generality let further be assumed simple and uniform values $\lambda_{ij_i}=1$, because these are out of the matrix inequality.

According to Schur Complement Lemma [21–23], matrix inequalities (17) can be turned into the solvable LMI and, if needed to involve some uncertainty factor, further alyzed by using Uncertainty Representation Lemma. Then the solution for gain matrices are computed with the LMI toolbox as follows:

$$K_{11}=[-0.131\quad -0.1148],$$
$$K_{12}=[-0.0623\quad -2.302],$$

$$K_{21}=[-4.4991\quad -2.4986],$$
$$K_{22}=[-5.4991\quad -3.4986],$$

$$P=\begin{bmatrix}0.0937 & 0.2146\\ 0.2146 & 0.6417\end{bmatrix}.$$

The design the switching law

$$\sigma(x)=\arg\min\{\bar{V}_i(x)\},$$

as emphasized in Theorem 1, in this example was emulated by means of the following formula:

$$\sigma(x)=\arg\min\{\bar{V}_i(x)\}\overset{\Delta}{=}\max_{j_i,\vartheta_i,p_i,q_i}\{$$
$$x^T\begin{bmatrix}(A_{ij_i}+B_{2ij_i}K_{i\vartheta_i})^T P+P(A_{ij_i}+B_{2ij_i}K_{i\vartheta_i})+\\ \frac{1}{\gamma^2}PB_{1ij_i}B_{1i\vartheta_i}^T P+(C_{ij_i}+D_{ij_i}K_{i\vartheta_i})^T\bullet\\ (C_{ip_i}+D_{ip_i}K_{iq_i})\end{bmatrix}x<0$$
$$j_i,\vartheta_i,p_i,q_i=1,2\}\}. \tag{18}$$

A selected set of graphically depicted simulation results using Matlab-Simulink platform [34–37] are presented in the sequel in order to demonstrate both the acting feasibility and achievable performance by the proposed control design synthesis in the closed loop. The simulation results are obtained by assuming plant system is at initial disturbed states to cold/chilly temperatures $[-3 \quad 0]^T$.

Time evolutions of the system states of this two-subsystem, two-dimensional system plant in four-rule Takagi-Sugeno representation model the closed loop with initial conditions $[-3 \quad 0]^T$ and under the hybrid control law of the H_∞ control and switching based control (Lemma 1 and Theorem 1) are presented in Figure 1. Similarly the time-evolution of the effective acting switching time sequence is presented in Figure 2.

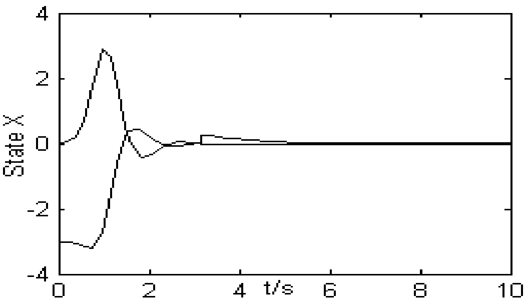

Fig. 1. The state response of the system with constructed overall PDC controller (5a)–(5b) according to Theorem 1 under switching law (8)

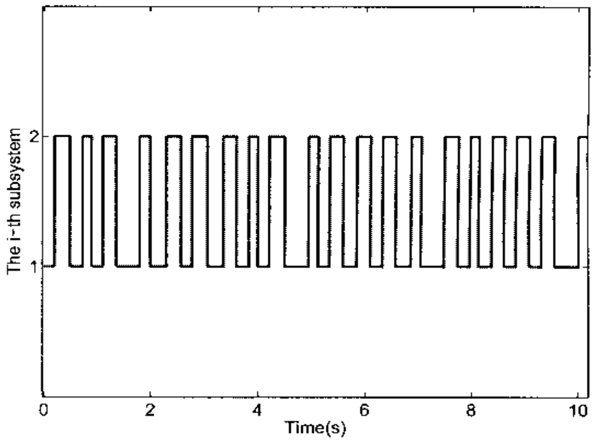

Fig. 2. The switching time sequence (8) that accompany the constructed PDC controller (5a)–(5b) according to Theorem 1

It should be noted, by the time $t = 10$ s, switching sequence exhibits almost periodically repeated switching jumps. Apparently, the state feedback H_∞ robust control problem with $\gamma = 1$ guarantying asymptotic stability is solved and the respetive two controls are depicted in Figure 3. Furthermore, in this particular plant example, it is interesting to notice that as if from the time $t = 10$ s onwards there appeared no more effective need for controlling actions.

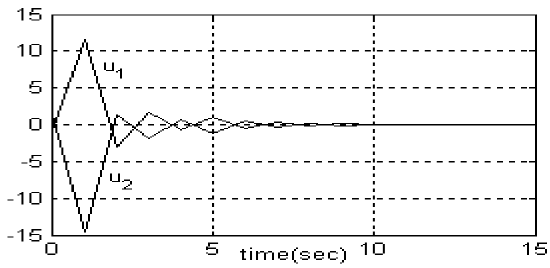

Fig. 3. The H_∞ state-feedback controls (5b) that accompany the constructed controller in PDC-architecture (5a) according to Theorem 1, and switching law (8); no further controlling activities are noticeable beyond 10 s whereas rather strong are during the first few seconds

Practically the equilibrium operating rise of controlled room temperature to the equilibrium state is achieved in finite time.

It is also interesting to note, in this traditional temperature control system but employing the hybrid H_∞ plus switching sequence in PDC-architecture of fuzzy-logic driven control certain additionnal highlights are obtained when subject to unknown impulsive Markov stochastic sudden disturbance hits on the actuators over longer period of operating time. At this point let recall the introduced Uncertainty Representation Lemma in the previous section. The found simulation results for feasible acting controls and achievable controlled states are depicted in Figures 4 and 5, respectively.

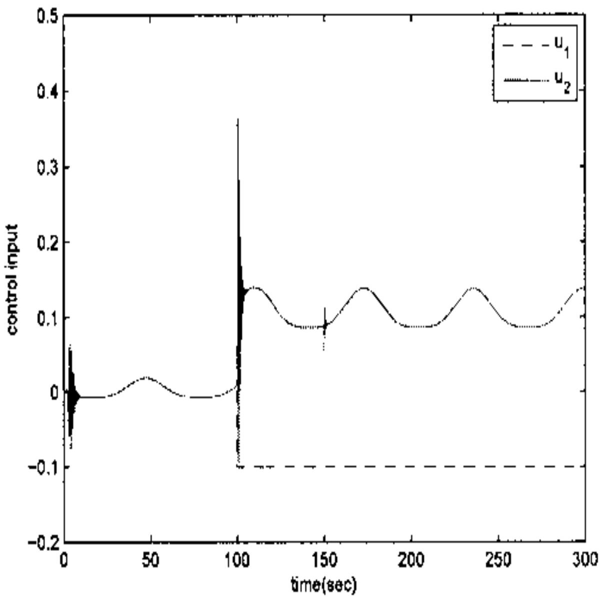

Fig. 4. Time history of the controlling activities under employing the proposed hybrid H_∞ plus switching sequence in PDC-architecture of fuzzy-logic driven control during a long operating time when it is being disturbed by sudden unknown Markov impulsive hits on actuating heaters

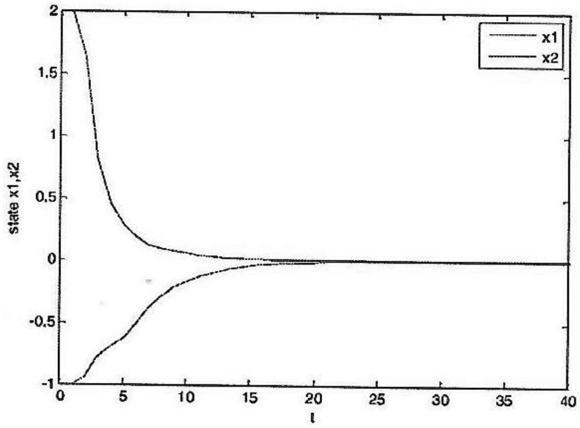

Fig. 5. Time history during the first 40 seconds of the controlled temperature state following Markov impulsive disturbance hits on actuating heaters; this same pattern repeats almost periodically following the operating controls.

5. CONCLUSION

The problem of state feedback H_∞ control for switched fuzzy systems is investigated via a non-traditional approach. In particular, considerable attention is focused on exploring switched fuzzy model involving implicit context of reliability, which has not been considered in previous studies even in context of reliable controls. The state space $\Omega \subseteq R^n$ of a switched system observed as a (fuzzy-)set partition $\Omega \subseteq \{\Omega_1, \cdots, \Omega_i, \cdots, \Omega_m\}$ into m sub-areas, thus every subarea emulating one switched subsystem. The orchestrated switching among subsystems via a purpose driven switching law design is aimed at ensuring stability of the overall switched system.

On the grounds of envisaged switching strategy, feedback controller and switching law of the state-dependent form are developed such that the problem of H_∞ control is solved. Sufficient condition for asymptotic stability based on Lyapunov theory is given. According to this condition, in order to check closed-loop stability a certain convex combination of subsystem matrices is to be checked, which is fairly easy. Simulation results for an application to real-world room air-conditioning illustrate both the effectiveness and feasible quality operating performance of this control design synthesis.

Acknowledgements: The authors acknowledge with gratitude the contributions made in due times by the respective doctoral students at Northeastern University of Shenyang in Shenyang, College of Information Science and Engineering, Liaoning, P. R. of China, at SS Cyril & Methodius University in Skopje, Faculty of Electrical Engineering & Information Technologies, N. Macedonia, with whom they have accomplished all research endeavors during last twenty years, this study inclusive.

Fig. 6. December 1996 in FEIT Laboratory of Electronics: Days 1996-97 at IASE-FEIT of fruitful postdoctoral research and visiting professorship by Yuan-Wei Jing (Systems & Control Theory to our students) in collaboration with Georgi Dimirovski

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