Article 222
Received: May 20, 2024
Accepted: November 10, 2024

Journal of Electrical Engineering and Information Technologies, Vol. **9**, No. 2 pp. 87-92 (2024)
In print: ISSN 2545–4250
Online: ISSN 2545–4269
UDC: 621.313.333.013.6
doi.org/10.51466/JEEIT2492222087t

Original Scientific Paper

THE IMPACT OF HIGH ORDER HARMONICS OF SUPPLY VOLTAGE ON OPERATING CHARACTERISTICS OF INDUCTION MOTORS

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Abstract: This paper considers the impact of high order harmonics on efficiency, power factor (PF), stator current, and electromagnetic torque developed by a three-phase induction cage motor. In that regard, three cases are examined in detail: the first - when the induction motor is connected to sinusoidal voltage, the second - when the induction motor is connected to voltage that contains high order harmonics (second and third order) despite the fundamental harmonic, and the third - when the induction motor is connected to voltage that contains both, i.e. fundamental and third order harmonic. Based on the simulation created in the Matlab SIMULINK programming and numeric platform, an analysis was made of each of the three cases, clearly highlighting the differences observed. The main aim of this paper is to make a comparison and estimate the error that could be made by neglecting the presence of one of the supply voltage high order harmonics for the induction motor, as well as to highlight the importance of taking these harmonics into account in calculations for operating characteristics of induction machines.

Keywords: high order harmonics, efficiency, induction motor

ВЛИЈАНИЕТО НА ВИШИТЕ ХАРМОНИЦИ НА ПРИКЛУЧНИОТ НАПОН ВРЗ РАБОТНИТЕ КАРАКТЕРИСТИКИ НА АСИНХРОНИТЕ МОТОРИ

Апстракт: Овој труд го разгледува влијанието на вишите хармоници врз коефициентот на полезно дејство, факторот на моќност, статорската струја и врз електромагнетниот момент што го развива трифазен асинхрон мотор со кафезен ротор. Притоа, детално се разгледани три случаи: еднаш - кога асинхрониот мотор е приклучен на чист синусен напон, вторпат - кога асинхрониот мотор е приклучен на напон кој содржи виши хармонци од втор и од трет ред и третпат - кога асинхрониот мотор е приклучен на напон кој го содржи основниот и третиот хармоник. Врз основа на симулацијата што е изработена во програмата Матлаб СИМУЛИНК, направена е анализа на секој од случаите, при што јасно се истакнати разликите што се забележуваат. Главната цел на овој труд е да се направи споредба и да се процени грешката што би се направила доколку во одредени случаи се занемари присуството на некој од вишите хармоници на приклучниот напон на асинхрониот мотор, како и да се истакне важноста од земањето предвид на овие хармоници во пресметките на работните карактеристики на моторите.

Клучни зборови: виши хармоници, ефикасност, асинхрон мотор.

I. INTRODUCTION

Induction motors are rotating electrical machines that should be used at rated frequency and sinusoidal form of voltage and load current. However, due to the increasing number of inverter-fed induction motors and feedback effects of the grid, there is no doubt that the analysis of the impact of high voltage and current harmonics on efficiency, power factor, and power losses of induction motors holds great importance. That is one of the main reasons why it is of crucial importance for significant harmonics to be taken into account when determining operating characteristics of induction motors [7].

In general, based on the direction of the magnetic fields produced by the harmonics, all harmonics occurring in the supply voltage can be divided into three groups [6]:

- positive;
- negative;
- zero sequence harmonics.

It should be noted that positive sequence harmonics (1,4,7, etc.) produce magnetic fields and currents rotating in the same direction as the fundamental frequency harmonic. On the other side, negative sequence harmonics (2,5,8, etc.) produce magnetic fields and currents that rotate in direction opposite to the fundamental frequency harmonic. Finally, zero sequence harmonics (3,9,15, etc.)

do not produce usable torque, but additional losses in the machine [12].

As a result of interaction between positive and negative sequence magnetic fields and currents, torsional oscillations of the motor shaft can be generated, leading to shaft vibrations that result in huge damage to the motor.

II. Mathematical Model Of Induction Motor

According to Fourier analysis, it is well-known that any non-sinusoidal waveform can be given as a sum of multiple sine waveforms with different frequency. In that respect, non-sinusoidal voltages and currents can be expressed as [10]:

$$v(t) = \sqrt{2}[V_1 \sin \omega_0 t + \sum_{k=2}^{\infty} V_k \sin(k\omega_0 t + \varphi_k)] \quad (1)$$

$$i(t) = \sqrt{2}[I_1 \sin \omega_0 t + \sum_{k=2}^{\infty} I_k \sin(k\omega_0 t + \theta_k)] \quad (2)$$

where:
V_1, I_1 are fundamental voltage and current;
V_k, I_k are k^{th} order harmonic voltage and current;
φ_k, θ_k are phase angles of k^{th} order harmonic voltage and current; and
ω_0, is radian frequency of the fundamental wave.

According to relevant IEEE standards, the total voltage harmonics distortion factor (THD-*U*) is defined as:

$$THD_U(\%) = \sqrt{\frac{\sum_{k=2}^{\infty} V_k^2}{V_1^2}} \times 100\% \quad (3)$$

and the amount of voltage distortion due to k^{th} order harmonic is measured by the voltage distortion factor (*VDF*) as [10]:

$$VDF(\%) = \frac{V_k}{V_1} \times 100\% \quad (4)$$

On the other side, the total current harmonics distortion factor (THD-*I*) is defined as:

$$THD_I(\%) = \sqrt{\frac{\sum_{k=2}^{\infty} I_k^2}{I_1^2}} \times 100\% \quad (5)$$

III. Simulink Model Of Induction Motor

In order to make qualitative and quantitative analysis of the impact of high order harmonics on operating characteristics of induction motors, a Simulink model was created, as shown in Figure 1.

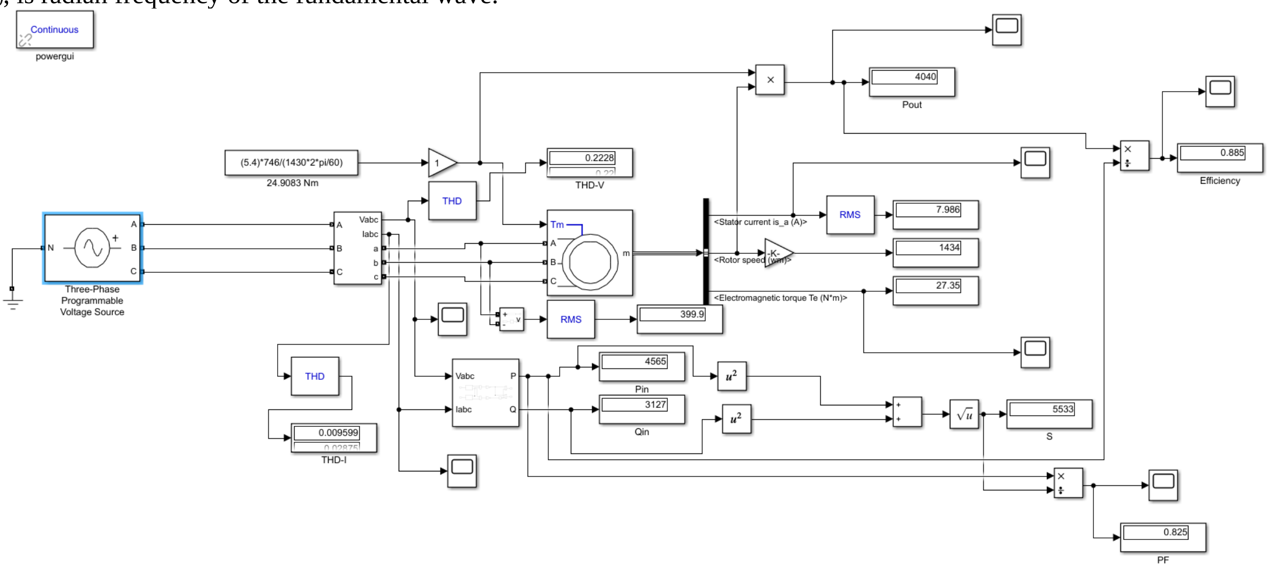


Fig. 1. Simulink model of induction motor

The analysis considered a three-phase induction squirrel-cage induction motor: P_n=4 kW, U_n=400 V, n_n=1430 rpm, f_n=50 Hz.

The block from Simscape library named "Three Phase Programmable Voltage Source" was used as a three-phase voltage source. This block implements a three-phase zero-impedance voltage source. Time variation for amplitude, phase, and frequency of the fundamental can be preprogrammed. In addition, two harmonics can be superimposed on the fundamental.

In order to measure the active and reactive power that the motor takes from the grid, a power block is also used, ensuring measurement of instantaneous active and reactive power.

It should be noted that each of the blocks is adapted to nominal parameters of the induction motor considered.

Table I provides an overview of general data for each of the harmonics taken into account during the analysis, such as amplitude of the harmonic in terms of the fundamental harmonic, its order, and sequence of the harmonics.

TABLE I
GENERAL DATA OF HARMONICS

Order of harmonics (h)	Amplitude (pu)	Phase (degrees)	Sequence
1	1	0	Positive
2	0.15	35	Negative
3	0.22	-25	Zero

IV. Results And Discussion

A. Total Harmonic Distortion

Based on the Matlab simulation of an induction motor, results were obtained for the total harmonic voltage and current distortion. Data presented in Table II show the values of the total harmonic voltage (THD-U) and current (THD-I) distortion, respectively.

TABLE II
TOTAL HARMONIC VOLTAGE AND CURRENT DISTORTION

Order of harmonics (h)	1	1 and 3	1,2 and 3
THD-U	0.0062	0.2228	0.2678
THD-I	$1.49 \cdot 10^{-8}$	0.009599	0.5724

It can be noted that the presence of harmonics significantly affects the total harmonic distortion, making it clear that they should not be neglected in calculations. The presence of both harmonics (2nd and 3th) greatly worsens the total harmonic distortion, which can be one of the main reasons for additional power losses that occur in induction machines. The value of the total voltage harmonic distortion when all harmonics are taken into account is 43.19 times higher than its value when they are neglected.

If the impact of the second harmonic is neglected, the total harmonic voltage distortion will be reduced by only 1.17 times, signifying that no significant error occurs.

B. Efficiency and Power Factor

TABLE III
EFFICIENCY AND POWER FACTOR (PF)

Order of harmonics (h)	1	1 and 3	1,2 and 3
Efficiency (%)	88.5	88.5	88.32
Power Factor (PF)	0.825	0.825	0.8264

It should be noted that this paper puts the emphasis on the impact of the second and third harmonics, and since harmonics of this order usually have a larger amplitude compared to harmonics of higher orders, it is clear that their influence cannot be neglected.

Values for the induction motor's efficiency and power factor for each of the cases considered are given in Table III. It can be seen that these values do not differ greatly for each of the cases considered, making it clear that they can be neglected in some cases.

C. Stator Current of Induction Motor

It is known that the fundamental component of the stator current is determined by the motor load and therefore it is clear that the relative harmonic content of the motor current is significantly higher at lower than at higher loads. This high harmonic content can cause a significant increase in power losses in unloaded machine, compared to its values when the machine is connected to sinusoidal voltage [12].

If V_k denotes the k^{th} harmonic component of the supply voltage, the corresponding stator current harmonic is $I_k=V_k/Z_k$, where Z_k is the k^{th} harmonic input impedance. For positive- and negative-sequence harmonics, $Z_k=k(X_1+X_2)$. Thus:

$$I_k = \frac{V_k}{k(X_1 + X_2)} \tag{6}$$

for zero-sequence harmonics, $Z_k=kX_0$; and

$$I_k = \frac{V_k}{kZ_0} \tag{7}$$

These formulas allow rapid evaluation of harmonic currents due to non-sinusoidal voltage waveform with known harmonic content. Usually, there are no zero-sequence harmonics and no even-numbered harmonics, thus the total rms harmonic current is given by [10]:

$$I_h = \sqrt{I_5^2 + I_7^2 + I_{11}^2 + I_{13}^2 + \ldots + I_k^2 + \ldots} \tag{8}$$

$$I_h = \sqrt{\sum_{k \neq 1} I_k^2} \tag{9}$$

If I_{rms} denotes the motor's fundamental root mean square (rms) current, it is clear that the total rms stator current, including the fundamental, can be calculated as [10]:

$$I_{rms} = \sqrt{I_1^2 + I_5^2 + I_7^2 + I_{11}^2 + I_{13}^2 + \ldots + I_k^2 + \ldots} \tag{10}$$

$$I_{rms} = \sqrt{I_1^2 + I_{har}^2} \tag{11}$$

It can be noted that, for a given voltage waveform, the relative harmonic content of the stator current is closely related to the motor's leakage reactance. In calculations and analyses, it is very practical to express the motor current, as well as the leakage reactance, in normalized or per-unit form (pu). They actually represent a ratio between the actual value of current or reactance and their base values.

The k^{th} harmonic current can be expressed in per-unit form as fraction of the rated full-load current. Thus:

$$I_k = \frac{V_k}{kX_{pu}} \tag{12}$$

where V_k is now the per-unit k^{th} harmonic voltage based on the rated sine wave voltage of the motor.

For operation at rated frequency, X_{pu} is the normal per-unit leakage reactance parameter of the motor, but obiously this reactance varies linearly with frequency. It is convenient to retain X_{pu} as the per-unit reactance at rated or base frequency and to take into account the frequency dependence of leakage reactance by means of a multiplying factor f_1, which is the per-unit fundamental frequency, and has value 1 at rated motor frequency. The k^{th} harmonic per-unit current at a per-unit fundamental frequency, f_1, is then given by:

$$I_k = \frac{V_k}{kf_1 X_{pu}} \tag{13}$$

For the six-step voltage waveform, the magnitude of each harmonic voltage is inversely proportional to the order of the harmonic. Thus:

$$V_k = \frac{V_1}{k} \tag{14}$$

and if this expression is substituted in equation (13), the following expression can be obtained:

$$I_k = \frac{V_1}{k^2 f_1 X_{pu}} \quad (15)$$

Fundamental airgap flux is directly proportional to stator induced electromotive force (emf), E_1, and inversely proportional to frequency. In order to obtain a good approximation, except at low frequency, airgap flux is therefore proportional to V/f. If the base value of airgap flux is that corresponding to the rated terminal voltage, V_R, at rated frequency, f_R, then the per-unit fundamental airgap flux can be expressed as:

$$\Phi_1 = \frac{V_1}{f_1} \quad (16)$$

where V_1 and f_1 are also expressed in per-unit form.

If the voltage V_1 is expressed from equation (16) and if this value is substituted in equation (15), the kth harmonic of the current can be obtained:

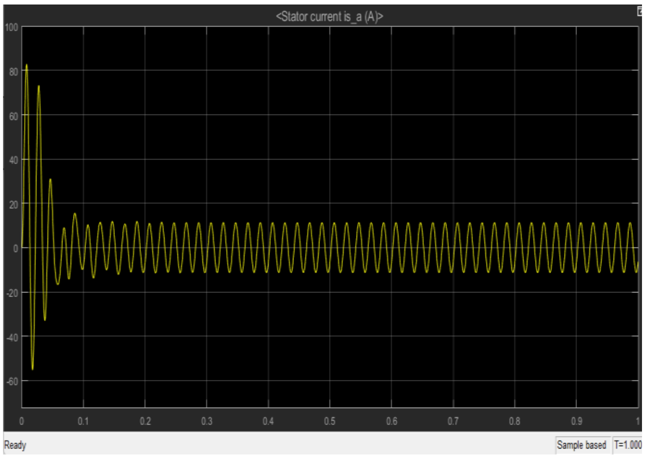
$$I_k = \frac{\Phi_1}{k^2 X_{pu}} \quad (17)$$

At normal conditions, i.e. with constant volt-hertz regulation, the base frequency Φ_1 is unity, so now the equation can be written as:

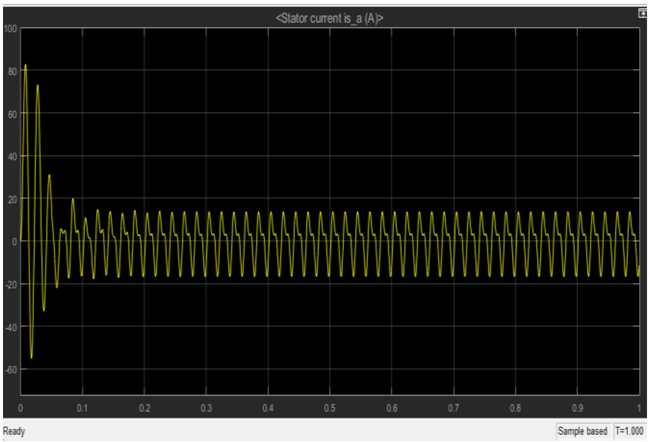
$$I_k = \frac{1}{k^2 X_{pu}} \quad (18)$$

Since X_{pu} is the per-unit reactance at base frequency, it is evident that harmonic current amplitudes are independent of supply frequency and motor load if the volts/hertz ratio is constant.

Figure 2 shows the stator current flowing through one of the phase windings. Figure 2a shows the waveform of the stator current when high order harmonics are neglected. On the other side, Figure 2b shows the stator current waveform when high order harmonics are taken into account. At the same time, it should be noted that the current largely deviates from its sinusoidal shape in the cases when the harmonics of the supply voltage are not neglected, which leads to increased power losses in the induction motor's windings. Based on the graphic, it can also be observed that the initial value of the motor inrush current is almost no different for each of the cases considered.



(a)



(b)

Fig. 2. Stator current of induction motor: a) when high order harmonics are neglected, b) when high order harmonics are not neglected

D. Electromagnetic Torque

On the other hand, Figure 3 shows the electromagnetic torque developed by the induction motor. Figure 3a shows the electromagnetic torque assuming higher harmonics are neglected, while Figure 3b shows the electromagnetic torque of the induction motor taking into account higher harmonics.

It can be seen that in the cases when the higher harmonics are not neglected, the torque gets a constant value, but this is not the case when the high order harmonics are present [12].

The presence of time harmonic magnetomotive force (mmf) waves in the airgap results in additional harmonic torques on the rotor. These torques can be divided into two groups: steady harmonic torques and pulsating harmonic torques.

Constant or steady torques are developed by the reaction of harmonic airgap fluxes with harmonic rotor mmfs, or currents, of the same order. However, these steady harmonic torques, which are a very small fraction of rated torque, have negligible effect on motor operation. This may be verified by calculating the torque contribution from the harmonic equivalent circuit, just as fundamental torque is derived from the fundamental equivalent circuit. Thus, the fundamental torque is given as:

$$T_1 = \frac{pm_1}{2\pi f_1} I_2^2 \frac{R_2}{s_1} \quad (19)$$

Similarly, based on the corresponding electrical equivalent circuits for each of the harmonics, the following expression can be rewritten for the kth harmonic of the moment:

$$T_k = \pm \frac{pm_1}{2\pi k f_1} I_{2k}^2 \frac{R_{2k}}{s_k} \quad (20)$$

where forward torque due to a positive-sequence harmonic is positive and backward torque due to a negative-sequence harmonic is negative.

The fundamental slip, s_1, is small for normal full-load operation of the induction motor, and consequently the following expression can be written for the harmonic slip [12]:

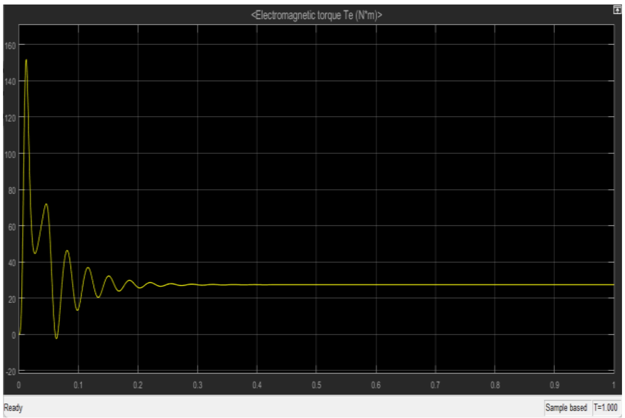
$$s_k = \frac{k \mp 1}{k} \quad (21)$$

Substitution in equation (20) gives the following:

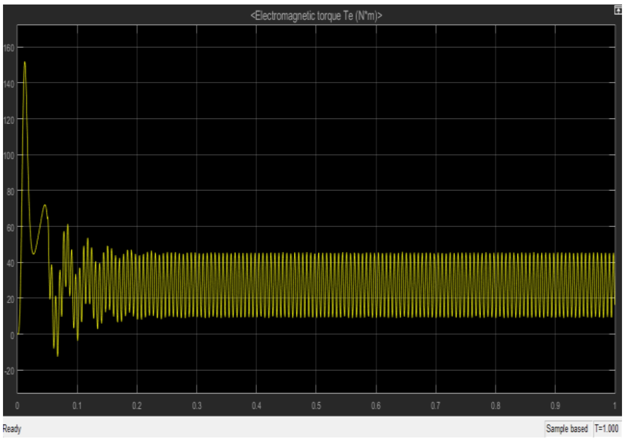
$$T_k = \pm \frac{pm_1}{2\pi f_1} I_{2k}^2 \frac{R_{2k}}{k \mp 1} \quad (22)$$

Thus, the k^th^ harmonic torque, expressed as a fraction of the fundamental torque, is:

$$\frac{T_k}{T_1} = \pm \left(\frac{I_{2k}}{I_2}\right)^2 \left(\frac{R_{2k}}{R_2}\right)\left(\frac{s_1}{k \mp 1}\right) \quad (23)$$



(a)



(b)

Fig. 3. Electromagnetic torque: a) when high order harmonics are neglected, b) when high order harmonics are not neglected

If the motor operates at rated load, then I_2 is near rated current and (I_{2k}/ I_2) is approximately equal to the per-unit k^th^ harmonic of the current, as given in equation (13). Substituting this expression in equation (23) gives the general equation:

$$\frac{T_k}{T_1} = \pm \left(\frac{V_k}{kf_1 X_{pu}}\right)^2 \left(\frac{R_{2k}}{R_2}\right)\left(\frac{s_1}{k \mp 1}\right) \quad (24)$$

In the case of a six-step voltage waveform, the per-unit value of I_k, is given by equation (17), and substituting I_{2k}/I_2, in equation (23) gives the corresponding harmonic torque equation as:

$$\frac{T_k}{T_1} = \pm \left(\frac{\Phi_1}{k^2 X_{pu}}\right)^2 \left(\frac{R_{2k}}{R_2}\right)\left(\frac{s_1}{k \mp 1}\right) \quad (25)$$

where Φ_1 is the per-unit airgap flux, as defined in equation (16).

Now consider the fifth harmonic torque in the case of an induction motor operating on a six-step voltage supply. Rated fundamental voltage and frequency are applied so that Φ_1 is unity.

Let it be assumed that the resistance of the rotor winding is three times greater as a result of the skin effect, while the slip value of the fundamental harmonic is 0.03. The relation between the moment for the fifth and the fundamental harmonic can be written as:

$$\frac{T_5}{T_1} = \frac{-0{,}24 \cdot 10^{-6}}{X_{pu}^2} \quad (26)$$

Thus, for a leakage reactance of 0.1 pu, it can be shown that T_5 is 0.24 percent of the fundamental torque. For a leakage reactance of 0.2 pu, T_5 is only 0.06 percent of T_1. This small counter-torque due to the negative-sequence fifth harmonic is opposed by a somewhat smaller forward torque due to the positive-sequence seventh harmonic. The combined effect of the fifth and seventh harmonics is, therefore, to produce a very small negative torque opposing the fundamental motor torque.

On the other side, the pulsating torques are produced as result of interaction between mmfs for each of the harmonics of the rotor and rotating magnetic fluxes for the harmonics of different order. Since the magnetic fluxes in the air gap for each of the harmonics have small values, it is clear that the dominant pulsating moments are actually the result of interaction between the rotor currents (or magnetomotive forces) and the fundamental rotating magnetic flux. For example, the fifth harmonic of the stator currents forms a system of inverse order and, at the same time, for the fundamental harmonic, creates a space wave of the magnetic voltage that rotates at a speed five times greater than the fundamental synchronous speed, but in the direction opposite to that of the fundamental magnetic field.

The rotor currents produced by this time harmonic of the magnetic field interact with the fundamental rotating magnetic field, creating a pulsating torque whose frequency is six times greater than the fundamental, as a result of the fact that the speed of the rotor mmf waveform for the corresponding harmonic in the air gap compared to the fundamental magnetic field in the airgap is actually six times greater than the synchronous speed of the fundamental field. The seventh harmonic of the stator current also creates a pulsating torque whose frequency is six times greater than the fundamental. The order of the seventh harmonic is direct (positive) and therefore this harmonic creates a time harmonic of the rotating magnetic field with a seven times greater speed than the synchronous, in the same direction as that of the fundamental magnetic field.

Again, it can be noted that the relative velocity of the mmf waveform for the corresponding harmonic compared to the fundamental magnetic field in the air gap is six times greater than the synchronous velocity, but this time in the opposite direction. It can be concluded that the mean value of the pulsating torques is 0, but their presence creates an angular velocity of the rotor that changes during rotation. At very low speeds, motor rotation occurs in interrupted or stepped motions, and this cannot be tolerated in some applications.

When some high order voltage harmonics are applied to an induction motor, it can result in production of pulsating torque as a result of interaction between the harmonics and the motor's rotor slots. There is no doubt that the presence of non-sinusoidal voltage component can impact smooth rotation of the induction motor and this should not be neglected during construction of induction motors.

V. Conclusion And Future Work

Based on results and facts presented, it is not difficult to conclude that high order harmonics have a huge impact on several parameters of induction motors. That is one of the main reasons that sometimes lead to a number of possible side effects, such as mechanical damage or aging of induction motors. The extent to which these damages occur can be subject of further analysis and design of a solution to address these problems. In the end, it should be highlighted that, in all cases when the results obtained from this paper will be taken into account, industrial companies would clearly pay attention to the process of selecting the right electrical motor for their applications, thus increasing both economy and positive effect on the environment.

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