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OPTIMIZATION OF COMPLEX ENERGY SYSTEMS*

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Abstract: According to the common practice different energy systems are analyzed separately, without taking into consideration their mutual dependence. The goal of this paper is to illustrate the modeling and optimization of complex systems, i.e. multiple-energy carrier systems, by using the energy hub methodology. A multiple-energy carrier system consists of different energy infrastructures and serves various types of energy demands, such as electricity, heat etc. The energy hub concept is thus implemented in the formulation of the economic dispatch problem for a complex energy system. Moreover, the paper contains a linear optimal power flow formulation of a complex system with multiple energy hubs interconnected with the power grid. The analysis will be conducted over simply structured systems with the aim of illustrating the idea of integrated modeling and the comparison of the system's operating points obtained by separate and integrated optimization.

Key words: integrated modeling and analysis; cogeneration; distributed generation

ОПТИМИЗАЦИЈА НА КОМПЛЕКСНИ ЕНЕРГЕТСКИ СИСТЕМИ

Апстракт: Според стандардната практика различните енергетски системи се анализираат одвоено, без притоа да се води грижа за нивната меѓусебна зависност. Овој труд има за цел да ги прикаже моделирањето и оптимизацијата на комплексни енергетски системи со помош на методот на општ енергетски јазол. Притоа за комплексен се смета секој енергетски систем кој опфаќа повеќе енергетски инфраструктури и со кој се задоволува потрошувачката на различни видови енергија, на пример електрична, топлинска итн. Дефиниран е проблемот на економски диспечинг на комплексен енергетски систем моделиран со општ енергетски јазол. Исто така е претставена оптимизацијата на повеќе општи енергетски јазли меѓусебно поврзани со електрична мрежа која е моделирана со DC моделот. Анализата е спроведена врз едноставни системи, а послужи за концептуален приказ на интегрираното моделирање и споредба на работните режими добиени со одвоена и интегрирана оптимизација.

Клучни зборови: интегрирано моделирање и анализа; когенерација; дистрибуирано производство

1. INTRODUCTION

In confronting the task of energy sector decarbonization, we are, unsurprisingly, faced with obstacles such as large share of heating and cooling energy [1], diversity and innovation of new technologies, all of which shed light on the need of broad, interdisciplinary analyses. Contrary to past practices when the power [2] and the natural gas [3] systems where analyzed separately, today great efforts are being put into eliminating such cross-sectoral borders. For instance, the authors of [4] have compared a number of technologies in terms of their capability to facilitate the integration of renewable energy sources. Among them are small cogeneration plants, heat pumps, electric water heaters, electrolysis base technologies, electric vehicles etc. Evidently, all of them act as links between different energy systems.

On that accord, a large body of literature deals with the benefits from coordinated planning and operation of the energy systems ([5], [6]). In

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[7], for example, a detailed model has been proposed for a simultaneous calculation of the optimal power flows in the power and natural gas grids, while taking into consideration their mutual dependence. The energy hub concept presented in [8], [9] and [10] builds upon and broadens this approach, thus allowing for an arbitrary number of different energy systems to be modeled and optimized.

The mathematical model proposed in the energy hub methodology can be utilized for defining many of the optimization problems which have previously been known and studied for the electrical power system. The reader is referred to [11] and [12] for a detailed overview of the optimal dispatching and complete optimal power flow formulations for multi-carrier energy systems. Analogous to the problem of unit commitment of power generation units, the authors of [13] have formulated the unit commitment problem for multi-carrier energy systems and have expanded its scope by introducing energy storage technologies in [14]. On the other hand, the energy hub concept has also been suited for systems with distributed renewable based generation of electricity [15]. Similarly, a predictive control of multiple energy hubs has been proposed in [17], while a simplified, linear model of the power grid has been introduced in [18] and has been used alongside the well-known nonlinear equations used to determine the flows in the gas grid.

The purpose of this paper is to illustrate the formulation of the optimization problems in complex (multi-carrier) energy systems and to study their effects on the total operation costs. Such systems have been modeled using the energy hub principle; the optimal dispatch problem is discussed and a DC optimal power flow has been proposed in order to further simplify and linearize the model.

2. METHODOLOGY

2.1. *Energy hub*

The energy hub is defined as a mathematical, fictitious structure in which different energy infrastructures meet. It establishes a relationship between what is injected as its input and the energy consumption it supplies. Let us use $\alpha, \beta, \ldots, \omega \in E =$ {electricity, heat, natural gas, biomass…} to denote the various energy carriers injected in the hub. Then $P_\alpha, P_\beta, \ldots, P_\omega$ shall be used to denote the corresponding powers injected from $\alpha, \beta, \ldots, \omega$, while $L_\alpha, L_\beta, \ldots, L_\omega$ shall be used to describe the energy loads. If a system has multiple energy hubs, each hub is enumerated as $i, j, k \ldots \in H = \{1, 2, 3, \ldots, N_H\}$.

The elements of the energy hub, the converter elements and the storage elements, determine the manner by which the loads, represented by the load vector $\boldsymbol{L}$, shall be satisfied by the hubs injections $\boldsymbol{P}$. The converter elements have an arbitrary number of input/output connections and are used to transform power into other forms or qualities (Figure 1). Examples of converter elements which qualitatively transform energy are CHP plants, motors, heat pumps, boilers etc. However, elements such as overhead lines, power cables, transformers, power electronics and compressors can also be defined as converter elements. They do not change the form of the energy, but transform some of its working parameters. Energy storage elements do not fall into the scope of this paper as they are of interest in time-series analysis and not analysis of single snapshots in time.

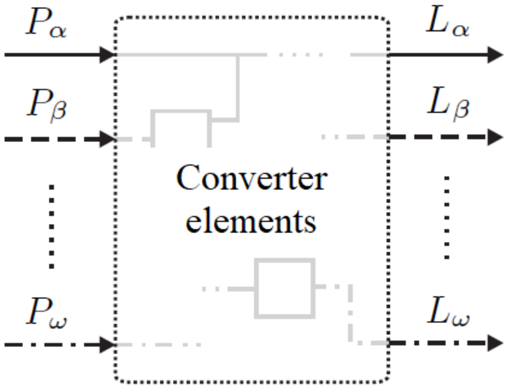

Fig. 1. General energy hub structure

For a single-input single-output converter element, the steady-state input P_α and the output L_β are correlated by the following equation:

$$L_\beta(k) = c_{\alpha\beta}(k) P_\alpha(k), \tag{1}$$

where $c_{\alpha\beta}$ is the coupling (conversion) factor between the input and output power. In general, it represents a function describing the efficiency of the converter unit and has a value in the range of $0 \le c_{\alpha\beta} \le 1$. It this paper we assume a constant value for the all coupling factors. When losses are neglected $c_{\alpha\beta} = 1$, while $c_{\alpha\beta} = 0$ if there is no converter element to couple α and β. When $\alpha = \beta$, the converter element doesn't qualitatively transform the input. The coupling factors of each converter element in the energy hub constitute the correlation matrix ***C***.

Each energy hub is fully determined by its input vector containing the injected powers ***P*** = [P_α, P_β,…, P_ω], the output vector containing the loads ***L*** = [L_α, L_β,…, L_ω] and the correlation matrix ***C***. The correlation matrix defines how the demand ***L*** will be supplied by the injected powers ***P*** through the converter elements. The internal flows in energy hub are thus given by:

$$\begin{bmatrix} L_\alpha \\ L_\beta \\ \vdots \\ L_\omega \end{bmatrix} = \begin{bmatrix} c_{\alpha\alpha} & c_{\beta\alpha} & \cdots & c_{\omega\alpha} \\ c_{\alpha\beta} & c_{\beta\beta} & \cdots & c_{\omega\beta} \\ \vdots & \vdots & \ddots & \vdots \\ c_{\alpha\omega} & c_{\beta\omega} & \cdots & c_{\omega\omega} \end{bmatrix} \begin{bmatrix} P_\alpha \\ P_\beta \\ \vdots \\ P_\omega \end{bmatrix} \tag{2}$$

Having in mind the conservation of energy law, the sum of the all coupling factors in one column of ***C*** should be less than or equal to one. If not specified otherwise, the energy flows are directed from the input towards the output of the energy hub.

2.2. *Relationship between injected powers and flows in the power grid*

For a power grid represented with the DC model which has *m* branches, *n* nodes and *ng* electric generators, the corresponding ***H*** matrix gives the relationship between the powers injected by the electric generators $\boldsymbol{P_{GEN}}$ and the power flows through the grid $\boldsymbol{P_{GR}}$ [19]:

$$\boldsymbol{P_{GR}} = \boldsymbol{H}\boldsymbol{P_{GEN}}\,. \tag{3}$$

The number of columns in the matrix is equal to the number of generators in the system, i.e. elements of $\boldsymbol{P_{GEN}}$. The formulation procedure of the full ***H*** matrix for all nodes and branches and the derivation of a reduced matrix of size *m* × *ng*, here used in equation (3), is discussed in [20].

Let α denote electricity, while β, γ,...,ω are used for all elements corresponding to other energy carriers. The flow in the power grid in a multi-carrier energy system with N_H energy hubs is calculated by the equation:

$$\boldsymbol{P_{GR}} = \begin{bmatrix} \boldsymbol{H_\alpha} & \boldsymbol{H_\beta} & \cdots & \boldsymbol{H_\omega} \end{bmatrix} \begin{bmatrix} \boldsymbol{P_\alpha} \\ \boldsymbol{P_\beta} \\ \vdots \\ \boldsymbol{P_\omega} \end{bmatrix}. \tag{4}$$

The matrix $\boldsymbol{H_\alpha}$ takes into account the generators which are not modeled as converter elements and are outside of the energy hub structure. This matrix is equal to the ***H*** matrix obtained from the [19]. Furthermore, the matrices $\boldsymbol{H_\beta}$, $\boldsymbol{H_\gamma}$,..., $\boldsymbol{H_\omega}$ deal with the electrical generators modeled as converter elements in the energy hubs and are calculated by the following steps:

- From the full ***H*** matrix of size *m* x *n* leave only those columns that not correspond to nodes (energy hubs) in which electrical generators modeled as converter elements are connected. Each of these matrices should end up having as many columns as there are converter elements of that energy carrier.
- Multiply column *i* of this matrix, corresponding to the converter element in the *i*-th node (energy hub), by the efficiency of that converter element.

The number of rows in the vectors $\boldsymbol{P_\alpha}$, $\boldsymbol{P_\beta}$,..., $\boldsymbol{P_\omega}$ is equal to the number of columns in the corresponding $\boldsymbol{H_\alpha}$, $\boldsymbol{H_\beta}$,..., $\boldsymbol{H_\omega}$.

3. OPTIMIZATION

3.1. *Economic dispatch*

The economic dispatch problem is presented for a multi-carrier energy system modeled as an energy hub. The optimization output is a vector ***P*** which satisfies the technical limitations of the generators, meets the energy demands of the loads and gives a global extreme for the objective function.

If the operational costs of each generator are represented by quadratic functions of the output power, the objective function *F*(***P***) can be defined as:

$$F(\boldsymbol{P}) = \sum_{i\in\{\alpha,\beta,\ldots,\omega\}} \left(a_i + b_i P_i + c_i P_i^2\right). \tag{5}$$

The mathematical formulation of the economic dispatch problem of a multi-carrier energy system is thus given by:

$$\min F(\boldsymbol{P}) \tag{6}$$

subject to

$$\boldsymbol{L} - \boldsymbol{C}\boldsymbol{P} = 0 \tag{7}$$

$$\boldsymbol{P_{min}} \le \boldsymbol{P} \le \boldsymbol{P_{max}} \tag{8}$$

where (7) defines the internal energy flows of the hub, while (8) describes the technical limitations of the generators regarding their minimum and maximum outputs.

3.2. *Optimal power flow in a multi-carrier energy system with a power grid represented with the DC model*

Let us analyze a multi-carrier energy system with N_H energy hubs interconnected with a power grid represented with the DC model. Firstly, we must assume that the error introduced by the DC model is negligible and secondly, we must know the allowed range for the natural gas injections in each hub. This allows us to explicitly calculate the electric power flows and approximate the state of the natural gas grid.

The optimal power flow of a multi-carrier energy system with a power grid represented by the DC model can therefore be defined as:

$$\min F(\boldsymbol{P}) \tag{9}$$

subject to:

$$\boldsymbol{L}_i - \boldsymbol{C}_i \boldsymbol{P}_i = 0 \tag{10}$$

$$\begin{bmatrix} \boldsymbol{H}_\alpha & \boldsymbol{H}_\beta & \cdots & \boldsymbol{H}_\omega \\ -\boldsymbol{H}_\alpha & -\boldsymbol{H}_\beta & \cdots & \boldsymbol{H}_\omega \end{bmatrix} \begin{bmatrix} \boldsymbol{P}_\alpha \\ \boldsymbol{P}_\beta \\ \vdots \\ \boldsymbol{P}_\omega \end{bmatrix} < \begin{bmatrix} \boldsymbol{P}_{\boldsymbol{GR}}^{\max} \\ \boldsymbol{P}_{\boldsymbol{GR}}^{\max} \end{bmatrix} \tag{11}$$

$$\boldsymbol{P}_i^{\min} \le \boldsymbol{P}_i \le \boldsymbol{P}_i^{\max} \tag{12}$$

where $i = \{1, 2, 3, \ldots, N_H\}$ stands for the i-th energy hub. Equation (10) describes the internal energy flows of each hub the inequality (11) represents the power flows in the grid. The limitations of each generator in all energy hubs are taken into consideration by (12).

4. RESULTS AND DISCUSSION

Two examples have been used to analyze the optimization problems from Section 3. The first example has been used to compare the results obtained from an integrated and a separate optimization of a complex energy system. The second example depicts a multi-carrier energy system used to illustrate the linear optimal power flow formulation. The modelling and simulations have been completed using the MATLAB programming language.

4.1. *A comparative analysis of integrated and separate optimization*

This section provides the results from the analyses of System I, given in Appendix A. The economic dispatch problem has been solved six times, i.e. for each of the two load scenarios, Scenario A and Scenario B, the system has been optimized:

- separately (*E→H approach*),
- separately (*H→E approach*),
- as an integrated system.

For a credible representation of the CHP's role, the separate optimization has been solved by two different approaches. The first approach (*E→H*) considers solving the economic dispatch for the units which generate electricity first. Afterwards, the heat produced by the CHP as a byproduct of the optimization is subtracted from the total heat load and the economic dispatch problem is solved again for the remaining units which generate heat. The second approach (*H→E*) analyzes the problem from the opposite direction, by solving the economic dispatch for the heat producing units before optimizing the units which generate electricity. Tables 1 and 2 show the total costs F of the multi-carrier energy system, the costs for supplying the electricity load F_e and the costs for supplying the heat load F_h for each analysis. The injections had been expressed in relative units (pu), while the costs are given in monetary units (mu).

Table 1

Results from Scenario A

	F_e (mu)	F_h (mu)	F (mu)
E→H	54.65	47.49	102.14
H→E	54.65	47.49	102.14
Integrated	54.95	43.19	98.15

Table 2

Results from Scenario B

	Fe (mu)	*Fh* (mu)	*F* (mu)
$E \rightarrow H$	101.78	19.71	121.49
$H \rightarrow E$	102.22	24.16	126.38
Integrated	97.00	19.23	116.23

The results show that when the loads are equal to $L_e = 4$ pu and $L_h = 7.5$, the order of separate optimization doesn't influence the outcome. One should note that this is not a general conclusion, but happens to hold for this specific load profile. In general, the order of optimization may influence the outcome. As evidence we provide the results from the load scenario $L_e = 7.5$ pu and $L_h = 4$ pu. The total costs obtained from a $E \rightarrow H$ and $H \rightarrow E$ optimization differ. Nevertheless, the obtained total costs F are the lowest when the electricity and heat systems are optimized as an integrated whole. Surely, this isn't far from expected. The integrated optimization of this system contains six variables subject to constraints in the form of two linear equations and inequalities determining the allowed range of each variable. On the other hand, when the electricity and heat systems are optimized separately, the heat byproduct produced by the CHP in the $E \rightarrow H$ approach, as well as the electricity byproduct of the CHP in the $H \rightarrow E$ approach impose an additional constraint, narrowing the search space and increasing the value of the objective function.

4.2. *Linearized optimal power flow*

System II of Appendix B consists of four energy hubs connected by the power grid, each containing a cogeneration unit (CHP) and a natural gas boiler (F). When the power grid is represented by the DC model and the OPF formulation of Section 3.2 is applied, the results shown in tables 3, 4 and 5 are obtained.

Table 3

Branch flows in the power grid

i – j	*PGR* (pu)
1 – 2	0.315
1 – 3	0.281
2 – 3	– 0.033
2 – 4	0.100

Table 4

Injections in CHP unit and natural gas boiler

i	*v*	*PCHP* (pu)	*PF* (pu)
1	0.952	1.143	0.057
2	0.010	0.007	0.663
3	0.010	0.007	0.663
4	0.556	0.500	0.400

Table 5

Generated electricity and heat

	Electricity		Heat	
i	P_G (pu)	P_{CHPe} (pu)	P_{CHPh} (pu)	P_{Fh} (pu)
1	0.503	0.343	0.457	0.043
2	0.000	0.002	0.003	0.497
3	–	0.002	0.003	0.497
4	–	0.150	0.200	0.300

The total operating costs of the system for this analyzed scenario are $F(\boldsymbol{P}) = 25.825$ mu. Table 4 shows the amounts of the natural gas injected in the CHP and the boiler F. The first column contains the dispatch factor v which represents the share of natural gas injected in the CHP with regards to the total injection in the energy hub. Although the structure of the energy hubs and the connected loads are identical, the dispatch factor v differs and is specific for each hub as a results of its location in the grid.

Not with standing the smaller level of detail when compared to the optimal power flow formulation of [12], the presented method allows fast calculations and analyses of large energy systems.

5. CONCLUSION

This paper was intended to illustrate a simple, yet useful modeling and optimization technique for complex, multi-carrier energy systems. In that regard, two systems were analyzed, each with a basic enough structure that the results could quickly lead to some general conclusions. The workload could be summarized as follows:

- The economic dispatch problem was used for the comparison of an integrated and separate optimization of the electricity and heat sector.
- The linearized optimal power flow was used to analyze the constraints that the electrical grid imposes on the converter and generation units.

One can conclude that the energy hub could easily be applied to the formulations of optimization problems in complex, multi-carrier energy systems, as it takes into consideration their mutual interdependences and permits finding solutions with the lowest total system costs.

REFERENCES

[1] European Commission, *An EU Strategy on Heating and Cooling*, Brussels, 2016.

[2] Dommel, H. W., Tinney, W. F.: Optimal Power Flow Solutions. *IEEE Transactions on Power Apparatus and Systems,* vol. **87** (1968).

[3] Wong, P. J., Larson. R. E.: Optimization of Natural-Gas Pipeline Systems Via Dynamic Programming, *IEEE Transactions on Automatic Control*, vol. **13** (1968).

[4] Mathiesen, B. V., Lund. H.: Comparative analyses of seven technologies to facilitate the integration of fluctuating renewable energy sources, *IET Renewable Generation*, vol. **3** (2009).

[5] Lund, H., Moller, B., Mathiesen, B. V., Dyrelund, A.: The role of district heating in future renewable energy systems*, Energy*, vol. **35** (2010).

[6] Zhang, X., Shadidehpour, M,, Abusorrah. A.: Optimal Expansion Planning of Energy Hub with Multiple Energy Infrastructure, *IEEE Transactions on Smart Grid,* vol. **6** (2015).

[7] An, S., Li, Q., Gedra. T: Natural Gas and Electricity Optimal Power Flow, *IEEE Power Engineering Society, Transmission Distribution Conference*, 2003.

[8] Geidl, M.: *Integrated Modelling and Optimization of Multi-Carrier Energy Systems,* Ph.D. thesis, ETH Zurich, 2007.

[9] Hemmes, K., Zachariah, L., Geidl, M., Andersson. G.: Towards Multi-Source Multi-Product Energy Systems, *International Journal of Hydrogen*, vol. **32** (2007).

[10] Fevre-Perrod, P., Geidl, M., Klockl, B., Koeppel. G.: A Vision of Future Energy Networks, *Innagural IEEE PES 2005 Conference and Exposition in Africa*, 2005.

[11] Geidl, M., Andersson, G.: Optimal Power Flow of Multiple Energy Carrier, *IEEE Transactions on Power Systems*, vol. **22** (2007).

[12] Geidl, M., Andersson, G.: Optimal Power Dispatch and Conversion in Systems with Multiple Carriers, *Proc. of 15th Power Systems Computation Conference*, Liege, Belgium, 2005.

[13] Ramirez–Elizondo, L. M., Paap, G.: Unit Commitment in Multiple Energy Carrier Systems*, North American Power Symposium,* 2009.

[14] Ramirez–Elizondo, L. M., Velez, V., Paap, G.: A technique for Unit Commitment in Multiple Energy Carrier Systems with Storage, *9th International Conference on Environment and Electrical Engineering*, 2010.

[15] Schulze, M., Friedrich, L., Gautschi, M.: Modeling and Optimization of Renewables: Applying the Energy Hub Approach, *2008 IEEE International Conference on Sustainable Energy Technologies*, 2008.

[16] Arnold, M. J.: *On Predictive Control for Coordination in Multiple-Carrier Energy Systems*, Ph.D. thesis, ETH Zurich, 2011.

[17] Li, G., Kou, Y., Liang, L., Bie, Z.: Researches on Reliability Evaluation of Integrated Energy Systems Based on Energy Hub, *2016 China International Conference on Electricity Distribution*, 2016.

[18] Almassalkhi, M., Hiskens, I.: Optimization Framework for the Analysis of Large-scale Networks of Energy Hubs, *17th Power System Computation Conference,* Stockholm, 2011.

[19] Todorovski, M., Ackovski, R.: Reduction of PTDF Matrix and its Application in DC Optimal Power Flow, *International Transactions on Electrical Energy Systems*, vol. **25** (2015).

[20] Ачковски, Р.: *Доверливосӣ во ӣреноснаӣа мрежа,* Материјали за настава по предметот Доверливост во ЕЕС, Скопје, 2007.

APPENDIX

A) *System I*

The system consists of four energy hubs connected together with the power grid. The *i*-th energy hub has a CHP unit and a natural gas boiler (F) and supplies an electrical and heat demand equal to $L_{ie} = 0.5$ pu and $L_{ih} = 1$ pu, respectively.

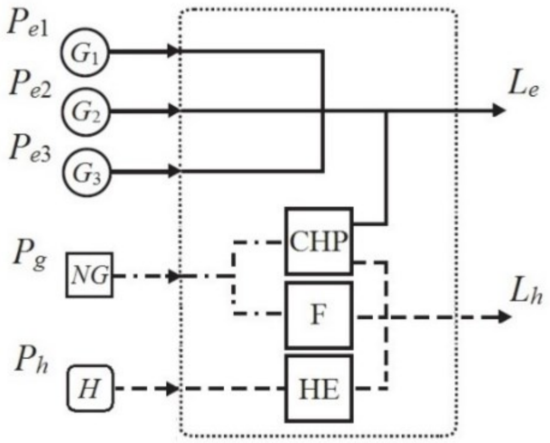

Fig. 2. System I

Table 6

Generator data

	$P_{\min}$ (pu)	$P_{\max}$ (pu)	*a* (mu)	*b* (mu·pu–1)	*c* (mu·pu–2)
*Pe*1	1.00	3.00	0.00	13.00	0.12
*Pe*2	0.50	2.00	0.00	13.00	0.13
*Pe*3	2.00	5.00	0.00	14.00	0.10
Pg	0.00	6.00	0.00	4.00	0.04
Ph	0.00	7.00	0.00	6.00	0.00

Table 7

Converter element efficiencies

ηCHPe	*ηCHPh*	*ηHE*	*ηF*
0.3	0.4	0.8	0.9

B) *System II*

The system consists of four energy hubs connected together with the power grid. The *i*-th energy hub has a CHP unit and a natural gas boiler (*F*) and supplies an electrical and heat demand equal to $L_{ie} = 0.5$ pu and $L_{ih} = 1$ pu, respectively.

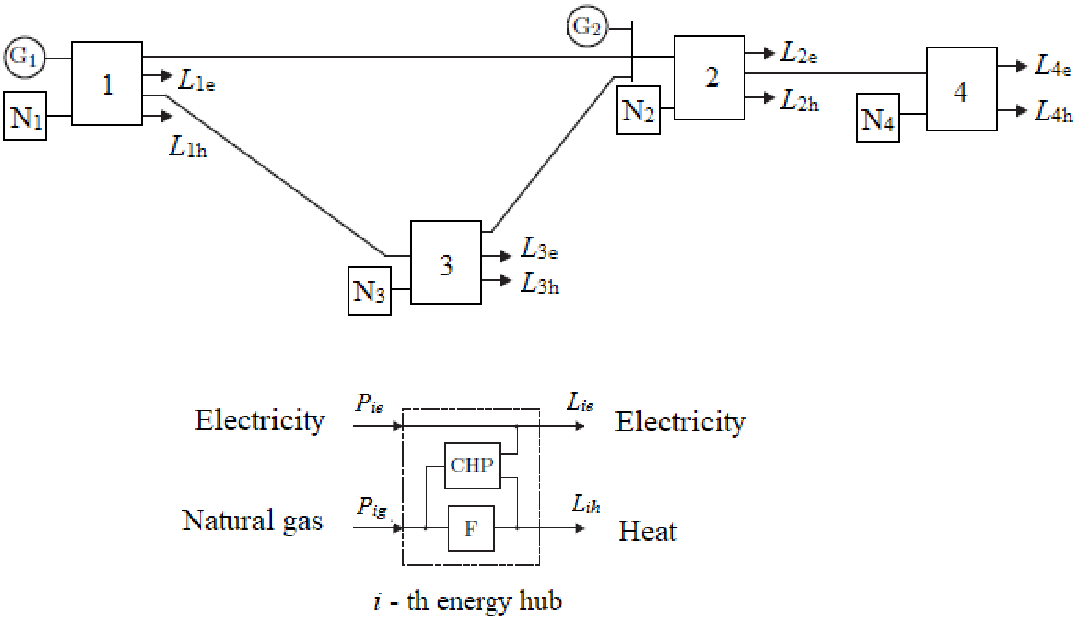

Fig. 3. System II

Table 8

Branch data

i – *j*	$X_{i\text{-}j}$ (pu)
1 – 2	0.90
2 – 3	0.90
2 – 3	0.90
2 – 4	0.90

Table 9

Generator cost function parameters

	a (mu)	*b* (mu·pu–1)	*c* (mu·pu–2)
*G*1	0,00	10.00	0.0010
*G*2	0.00	11.00	0.0011
*N*1	0.00	6.00	0.0600
*N*2	0.00	6.00	0.0600
*N*3	0.00	6.00	0.0600
*N*4	0.00	6.00	0.0600

Table 10

Converter element efficiencies

$\eta CHPe$	$\eta CHPh$	ηF
0.3	0.4	0.75

Table 11

Allowed range of natural gas injections in energy hubs

i	*Pig*min (pu)	*Pig*max (pu)
1	0.00	1.20
2	0.00	0.67
3	0.00	0.67
4	0.00	0.90